

Università degli Studi 'La Sapienza'

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Matematica Finanziaria

Canale D - K

Capitolo 3

Ammortamento di prestiti indivisi

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Capitolo 3 – Ammortamento di prestiti indivisi

3.1 – Concetti di base

Per ammortamento di un prestito (**mutuo**) indiviso si intende quel procedimento in base al quale un soggetto (unico) detto **mutuante** o creditore cede ad un tempo iniziale (es. 0) ad un altro soggetto detto **mutuatario** o debitore una somma D_0 (importo del prestito) a fronte della restituzione da parte del debitore di una rendita, finanziariamente equivalente alla somma prestata.

Tale procedimento di ammortamento è denominato progressivo e, considerato nel regime finanziario uniforme della capitalizzazione composta, risulta caratterizzato dalle seguenti grandezze:

- **Importo del prestito** D_0
- **Durata dell'ammortamento** n
- **Rate di ammortamento** $\begin{matrix} n \\ \square \\ k=1 \end{matrix} R_k$
- **Quote capitale** $\begin{matrix} n \\ \square \\ k=1 \end{matrix} C_k$
- **Quote interesse** $\begin{matrix} n \\ \square \\ k=1 \end{matrix} I_k$
- **Debiti residui** $\begin{matrix} n \\ \square \\ k=1 \end{matrix} D_k$

e dalle seguenti relazioni tra le diverse grandezze finanziarie:

- **Condizioni di equilibrio del processo di ammortamento (Debito iniziale = importo del prestito)**

$$D_0 = \begin{cases} \sum_{k=1}^n C_k \\ \sum_{k=1}^n R_k (1+i)^{-k} = \sum_{k=1}^n R_k v^k \end{cases}$$

- **Debiti residui (anche iniziale)**

$$\begin{cases} \prod_{k=0}^{n-1} D_k = \begin{cases} D_0 - \sum_{h=1}^k C_h = \sum_{h=k+1}^n C_h \\ D_0 r^k - \sum_{h=1}^k R_h r^{k-h} = \sum_{h=1}^n R_h v^{h-k} - \sum_{h=1}^k R_h v^{h-k} = \sum_{h=k+1}^n R_h v^{h-k} \end{cases} \\ D_n = 0 \end{cases}$$

- **Quote interessi**

$$\prod_{k=1}^n I_k = i D_{k-1} = \begin{cases} i \left(D_0 - \sum_{h=1}^{k-1} C_h \right) = i \sum_{h=k}^n C_h \\ i \left(D_0 r^{k-1} - \sum_{h=1}^{k-1} R_h r^{k-1-h} \right) = i \sum_{h=k}^n R_h v^{h-k+1} \end{cases}$$

- **Rate di ammortamento**

$$\prod_{k=1}^n R_k = C_k + I_k = C_k + i D_{k-1}$$

$$\prod_{k=1}^n C_k = R_k - I_k = R_k - i D_{k-1}$$

da cui possono trarsi diverse relazioni ricorrenti:

$$\prod_{k=1}^n D_k = \begin{cases} \sum_{h=k+1}^n C_h = \sum_{h=k}^n C_h - C_k = D_{k-1} - C_k \\ \sum_{h=k+1}^n R_h v^{h-k} = r \sum_{h=k+1}^n R_h v^{h-(k-1)} = r \left(\sum_{h=k}^n R_h v^{h-(k-1)} - R_k v \right) = r D_{k-1} - R_k \end{cases}$$

$$\prod_{k=1}^n C_k = D_{k-1} - D_k$$

$$\prod_{k=1}^n R_k = r D_{k-1} - D_k$$

$$\prod_{k=1}^n I_k = I_{k-1} - C_{k-1} i$$

$$\prod_{k=1}^n D_{k-1} = \begin{cases} D_k + C_k \\ (D_k + R_k)v \end{cases} \left(D_{n-1} = \begin{cases} C_n \\ R_n v \end{cases} \right)$$

$$\prod_{k=1}^n R_k = R_{k-1} + C_k - C_{k-1} r$$

3.2 – Ammortamenti con quote capitali prefissate

$$\prod_{k=1}^n C_k$$

$$\sum_{k=1}^n C_k = D_0$$

$$\prod_{k=1}^n D_k = D_{k-1} - C_k \dots = D_0 - \sum_{h=1}^k C_h = \sum_{h=k+1}^n C_h$$

$$\prod_{k=1}^n I_k = i D_{k-1} \dots = i \sum_{h=k}^n C_h$$

$$\prod_{k=1}^n R_k = C_k + I_k \dots = C_k + i \sum_{h=k}^n C_h$$

k	C_k	I_k	R_k	D_k
0				D_0
1	C_1	$I_1 = i D_0$	$R_1 = C_1 + I_1$	$D_1 = D_0 - C_1$
2	C_2	$I_2 = i D_1$	$R_2 = C_2 + I_2$	$D_2 = D_1 - C_2$
...
k	C_k	$I_k = i D_{k-1}$	$R_k = C_k + I_k$	$D_k = D_{k-1} - C_k$
...
n	C_n	$I_n = i D_{n-1}$	$R_n = C_n + I_n$	$D_n = 0$

Esercizio; Verificare che la somma dei valori attuali delle rate corrisponde all'importo del prestito

$$\begin{aligned}
 \sum_{k=1}^n R_k v^k &= \sum_{k=1}^n (C_k + I_k) v^k = \sum_{k=1}^n (C_k + i D_{k-1}) v^k = \sum_{k=1}^n (C_k + i \sum_{h=k}^n C_h) v^k = \\
 &= \sum_{k=1}^n C_k v^k + i \sum_{k=1}^n \sum_{h=k}^n C_h v^k = \sum_{k=1}^n C_k v^k + i \sum_{h=1}^n C_h \sum_{k=1}^h v^k = \\
 &= \sum_{k=1}^n C_k v^k + i \sum_{h=1}^n C_h \frac{1-v^h}{i} = \sum_{k=1}^n C_k v^k + \sum_{h=1}^n C_h - \sum_{h=1}^n C_h v^h = \sum_{h=1}^n C_h = D_0
 \end{aligned}$$

• **Ammortamento di mutuo puro**

(con quote capitali nulle, tranne l'ultima pari al debito iniziale)

$$\begin{aligned}
 \sum_{k=1}^{n-1} C_k &= 0 \\
 C_n &= D_0
 \end{aligned}$$

$$\begin{aligned}
 \sum_{k=1}^{n-1} D_k &= D_0, \quad D_n = 0 \\
 \sum_{k=1}^n I_k &= i D_0 \\
 \sum_{k=1}^{n-1} R_k &= I_k = i D_0, \quad R_n = D_0 + i D_0 = r D_0
 \end{aligned}$$

k	C_k	I_k	R_k	D_k
0				D_0
1	$C_1 = 0$	$I_1 = i D_0$	$R_1 = i D_0$	$D_1 = D_0$
2	$C_2 = 0$	$I_2 = i D_0$	$R_2 = i D_0$	$D_2 = D_0$
...
k	$C_k = 0$	$I_k = i D_0$	$R_k = i D_0$	$D_k = D_0$
...
n	$C_n = D_0$	$I_n = i D_0$	$R_n = r D_0$	$D_n = 0$

Esercizio : Verificare che la somma dei valori attuali delle rate corrisponde all'importo del prestito

$$\begin{aligned} \sum_{k=1}^n R_k v^k &= \sum_{k=1}^{n-1} i D_0 v^k + r D_0 v^n = i D_0 a_{\overline{n-1}|i} + D_0 v^{n-1} = \\ &= i D_0 \frac{1-v^{n-1}}{i} + D_0 v^{n-1} = D_0 (1-v^{n-1} + v^{n-1}) = D_0 \end{aligned}$$

• **Ammortamento di tipo italiano**

(con quote capitale costanti e rate in progressione aritmetica)

$$\underbrace{\sum_{k=1}^n C_k = \frac{D_0}{n}}_{\sum_{k=1}^n C_k = n \frac{D_0}{n} = D_0}$$

$$\begin{aligned} \sum_{k=1}^n D_k &= D_{k-1} - C \dots = D_0 - kC = (n-k)C = \frac{n-k}{n} D_0 \\ \sum_{k=1}^n I_k &= (n-k+1)iC, \quad \sum_{k=2}^n I_k = I_{k-1} - iC \\ \sum_{k=1}^n R_k &= (1+(n-k+1)i)C, \quad \sum_{k=2}^n R_k = R_{k-1} - iC \end{aligned}$$

k	C_k	I_k	R_k	D_k
0				D_0
1	$C_1 = C$	$I_1 = niC$	$R_1 = (1+ni)C$	$D_1 = (n-1)C$
2	$C_2 = C$	$I_2 = (n-1)iC$	$R_2 = (1+(n-1)i)C$	$D_2 = (n-2)C$
...
k	$C_k = C$	$I_k = (n-k+1)iC$	$R_k = (1+(n-k+1)i)C$	$D_k = (n-k)C$
...
n	$C_n = C$	$I_n = iC$	$R_n = (1+i)C$	$D_n = 0$

Esercizio: Verificare che la somma dei valori attuali delle rate corrisponde all'importo del prestito

$$\begin{aligned}
 \sum_{k=1}^n R_k v^k &= \sum_{k=1}^n \frac{1+(n-k+1)i}{n} D_0 v^k = \frac{D_0}{n} \sum_{k=1}^n (1+(n-k+1)i) v^k = \\
 &= \frac{D_0}{n} \left(\sum_{k=1}^n v^k + i \sum_{k=1}^n (n-k+1) v^k \right) = \frac{D_0}{n} \left(a_{\overline{n}|i} + i \left((n+1) \sum_{k=1}^n v^k - \sum_{k=1}^n k v^k \right) \right) = \\
 &= \frac{D_0}{n} \left(a_{\overline{n}|i} + i(n+1)a_{\overline{n}|i} - i(la)_{\overline{n}|i} \right) = \frac{D_0}{n} \left(a_{\overline{n}|i} + (n+1)(1-v^n) - \ddot{a}_{\overline{n}|i} + n v^n \right) = \\
 &= \frac{D_0}{n} \left(a_{\overline{n}|i} - \ddot{a}_{\overline{n}|i} + n+1 - n v^n - v^n + n v^n \right) = \frac{D_0}{n} (v^n - 1 + n+1 - v^n) = \frac{D_0}{n} n = D_0
 \end{aligned}$$

3.3 – Ammortamenti con rate prefissate

$$\begin{aligned}
 &\sum_{k=1}^n R_k \\
 &\sum_{k=1}^n R_k v^k = D_0
 \end{aligned}$$

$$\begin{aligned}
 \sum_{k=1}^n D_k &= r D_{k-1} - R_k \dots = D_0 r^k - \sum_{h=1}^k R_h r^{k-h} = \sum_{h=k+1}^n R_h v^{h-k} \\
 \sum_{k=1}^n I_k &= i D_{k-1} \dots = i \sum_{h=k}^n R_h v^{h-k+1} \\
 \sum_{k=1}^n C_k &= R_k - I_k \dots = C_k - i \sum_{h=k}^n R_h v^{h-k+1}
 \end{aligned}$$

k	C_k	I_k	R_k	D_k
0				D_0
1	$C_1 = R_1 - I_1$	$I_1 = i D_0$	R_1	$D_1 = D_0 - C_1$
2	$C_2 = R_2 - I_2$	$I_2 = i D_1$	R_2	$D_2 = D_1 - C_2$
...
k	$C_k = R_k - I_k$	$I_k = i D_{k-1}$	R_k	$D_k = D_{k-1} - C_k$
...
n	$C_n = R_n - I_n$	$I_n = i D_{n-1}$	R_n	$D_n = 0$

Esercizio: Verificare che la somma delle quote capitali corrisponde all'importo del prestito

$$\begin{aligned} \sum_{k=1}^n C_k &= \sum_{k=1}^n (R_k - I_k) = \sum_{k=1}^n (R_k - i D_{k-1}) = \sum_{k=1}^n (R_k - i \sum_{h=k}^n R_h v^{h-k+1}) = \\ &= \sum_{k=1}^n R_k - i \sum_{k=1}^n \sum_{h=k}^n R_h v^{h-k+1} = \sum_{k=1}^n R_k - i \sum_{h=1}^n R_h \sum_{k=1}^h v^{h-k+1} = \\ &= \sum_{k=1}^n R_k - i \sum_{h=1}^n R_h \frac{1-v^h}{i} = \sum_{k=1}^n R_k - \sum_{h=1}^n R_h + \sum_{h=1}^n R_h v^h = \sum_{h=1}^n R_h v^h = D_0 \end{aligned}$$

- **Ammortamento di tipo francese**

(con rate costanti e quote capitale in progressione geometrica)

$$\underbrace{\sum_{k=1}^n R_k}_{R} = \frac{D_0}{a_{\overline{n}|i}}$$

$$\sum_{k=1}^n R_k v^k = \frac{D_0}{a_{\overline{n}|i}} a_{\overline{n}|i} = D_0$$

$$\begin{aligned} \sum_{k=1}^n D_k &= r D_{k-1} - R \dots D_0 r^k - R s_{\overline{k}|i} = R a_{\overline{n-k}|i} = R \frac{a_{\overline{n-k}|i}}{a_{\overline{n}|i}} \\ \sum_{k=1}^n I_k &= i R a_{\overline{n-k+1}|i} = R(1-v^{n-k+1}), \quad \sum_{k=2}^n I_k = I_{k-1} - C_{k-1} i \\ \sum_{k=1}^n C_k &= R v^{n-k+1} = \frac{D_0}{s_{\overline{n}|i}} r^{k-1}, \quad \sum_{k=2}^n C_k = C_{k-1} r \end{aligned}$$

k	C_k	I_k	R_k	D_k
0				D_0
1	$C_1 = Rv^n$	$I_1 = R(1-v^n)$	$R_1 = R$	$D_1 = Ra_{\overline{n-1} i}$
2	$C_2 = Rv^{n-1}$	$I_2 = R(1-v^{n-1})$	$R_2 = R$	$D_2 = Ra_{\overline{n-2} i}$
...
k	$C_k = Rv^{n-k+1}$	$I_k = R(1-v^{n-k+1})$	$R_k = R$	$D_k = Ra_{\overline{n-k} i}$
...
n	$C_n = Rv$	$I_n = R(1-v)$	$R_n = R$	$D_n = 0$

Esercizio: Verificare che la somma delle quote capitali corrisponde all'importo del prestito e che le quote capitali sono in progressione geometrica di ragione r

$$\sum_{k=1}^n C_k = \sum_{k=1}^n \frac{D_0}{a_{\overline{n}|i}} v^{n-k+1} = \frac{D_0}{a_{\overline{n}|i}} v^n \sum_{k=1}^n r^{k-1} = \frac{D_0}{a_{\overline{n}|i}} v^n s_{\overline{n}|i} = \frac{D_0}{a_{\overline{n}|i}} a_{\overline{n}|i} = D_0$$

$$\prod_{k=2}^n R_k = R_{k-1} + C_k - C_{k-1} r \Rightarrow C_k = C_{k-1} r \text{ essendo } C_1 = \frac{D_0}{s_{\overline{n}|i}}$$

Esercizio

Dato un capitale P (es. 1000) prestato al tempo iniziale 0, costruire i piani di ammortamento relativamente ad una durata pari a T ($=5$) e ad un tasso effettivo annuo d'interesse i (es. 6%), nei diversi casi sotto indicati:

- **Caso 1: Ammortamento con quote capitali prefissate** (300, 100, 0, 400, ?)

$$C_n = D_0 - \sum_{k=1}^{n-1} C_k = 1000 - (300 + 100 + 0 + 400) = 200$$

	A	B	C	D	E
1	Cap	1000			
2	Anni	5			
3	Tax	6%			
4					
5	Anni	Capit	Inter	Rata	Residuo
6	0				1000.00
7	1	300.00	60.00	360.00	700.00
8	2	100.00	42.00	142.00	600.00
9	3	0.00	36.00	36.00	600.00
10	4	400.00	36.00	436.00	200.00
11	5	200.00	12.00	212.00	0.00
12		1000.00	186.00	1186.00	

	A	B	C	D	E
1	Cap	1000	.		
2	Anni	5			
3	Tax	0.06			
4					
5	Anni	Capit	Inter	Rata	Residuo
6	0				=B\$1
7	=A6+1	300	=E6*\$B\$3	=B7+C7	=E6-B7
8	=A7+1	100	=E7*\$B\$3	=B8+C8	=E7-B8
9	=A8+1	0	=E8*\$B\$3	=B9+C9	=E8-B9
10	=A9+1	400	=E9*\$B\$3	=B10+C10	=E9-B10
11	=A10+1	=E6-SOMMA(B7:B10)	=E10*\$B\$3	=B11+C11	=E10-B11
12		=SOMMA(B7:B11)	=SOMMA(C7:C11)	=SOMMA(D7:D11)	

- Caso 2: Ammortamento di mutuo puro**

$$\prod_{k=1}^{n-1} C_k = 0, C_n = D_0 = 1000$$

	A	B	C	D	E
15	Anni	Capit	Inter	Rata	Residuo
16	0				1000.00
17	1	0.00	60.00	60.00	1000.00
18	2	0.00	60.00	60.00	1000.00
19	3	0.00	60.00	60.00	1000.00
20	4	0.00	60.00	60.00	1000.00
21	5	1000.00	60.00	1060.00	0.00
22		1000.00	300.00	1300.00	

	A	B	C	D	E
15	Anni	Capit	Inter	Rata	Residuo
16	0				=B\$1
17	=A16+1	0	=E16*\$B\$3	=B17+C17	=E16-B17
18	=A17+1	0	=E17*\$B\$3	=B18+C18	=E17-B18
19	=A18+1	0	=E18*\$B\$3	=B19+C19	=E18-B19
20	=A19+1	0	=E19*\$B\$3	=B20+C20	=E19-B20
21	=A20+1	=E16	=E20*\$B\$3	=B21+C21	=E20-B21
22		=SOMMA(B17:B21)	=SOMMA(C17:C21)	=SOMMA(D17:D21)	

- Caso 3: Ammortamento di tipo italiano**

$$\sum_{k=1}^n C_k = C = \frac{D_0}{n} = \frac{1000}{5} = 200$$

	A	B	C	D	E
25	Anni	Capit	Inter	Rata	Residuo
26	0				1000.00
27	1	200.00	60.00	260.00	800.00
28	2	200.00	48.00	248.00	600.00
29	3	200.00	36.00	236.00	400.00
30	4	200.00	24.00	224.00	200.00
31	5	200.00	12.00	212.00	0.00
32		1000.00	180.00	1180.00	

	A	B	C	D	E
25	Anni	Capit	Inter	Rata	Residuo
26	0				=B\$1
27	=A26+1	=E26/B2	=E26*\$B\$3	=B27+C27	=E26-B27
28	=A27+1	=B27	=E27*\$B\$3	=B28+C28	=E27-B28
29	=A28+1	=B28	=E28*\$B\$3	=B29+C29	=E28-B29
30	=A29+1	=B29	=E29*\$B\$3	=B30+C30	=E29-B30
31	=A30+1	=B30	=E30*\$B\$3	=B31+C31	=E30-B31
32		=SOMMA(B27:B31)	=SOMMA(C27:C31)	=SOMMA(D27:D31)	

- **Caso 4: Ammortamento con rate prefissate (250, 180, 0, 400, ?)**

$$R_n v^n = D_0 - \sum_{k=1}^{n-1} R_k v^k = 1000 - (250v + 180v^2 + 400v^4), \quad R_n = D_{n-1} r$$

$$R_n = 1000r^5 - (250r^4 + 180r^3 + 400r) = 384.22 \dots = 362.47 \cdot (1+i)$$

	A	B	C	D	E
35	Anni	Capit	Inter	Rata	Residuo
36	0				1000.00
37	1	190.00	60.00	250.00	810.00
38	2	131.40	48.60	180.00	678.60
39	3	-40.72	40.72	0.00	719.32
40	4	356.84	43.16	400.00	362.47
41	5	362.47	21.75	384.22	0.00
42		1000.00	214.22	1214.22	

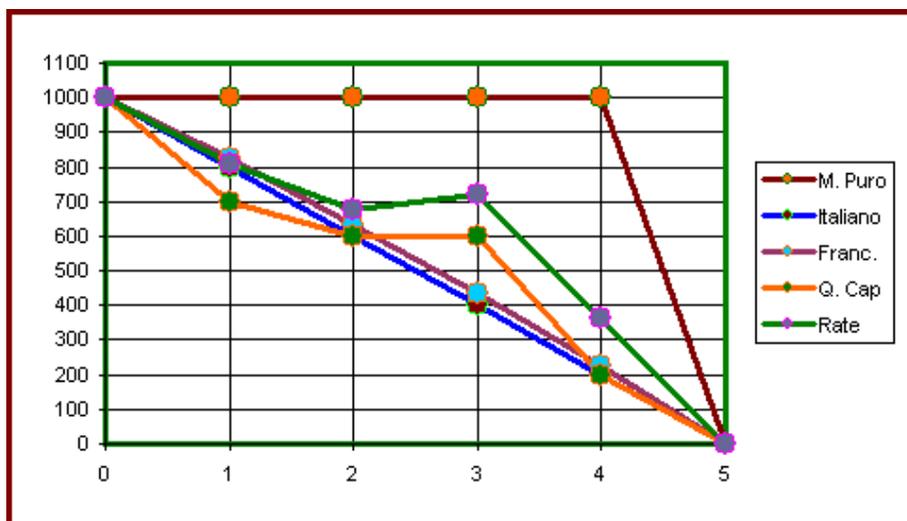
	A	B	C	D	E
35	Anni	Capit	Inter	Rata	Residuo
36	0				=B\$1
37	=A36+1	=D37-C37	=E36*\$B\$3	250	=E36-B37
38	=A37+1	=D38-C38	=E37*\$B\$3	180	=E37-B38
39	=A38+1	=D39-C39	=E38*\$B\$3	0	=E38-B39
40	=A39+1	=D40-C40	=E39*\$B\$3	400	=E39-B40
41	=A40+1	=D41-C41	=E40*\$B\$3	=E40*(1+B3)	=E40-B41
42		=SOMMA(B37:B41)	=SOMMA(C37:C41)	=SOMMA(D37:D41)	

• **Caso 5: Ammortamento di tipo francese**

$$\sum_{k=1}^n R_k = R = \frac{D_0}{a_{n|i}} = \frac{1000}{a_{5|0.06}} = 237.40$$

	A	B	C	D	E
45	Anni	Capit	Inter	Rata	Residuo
46	0				1000.00
47	1	177.40	60.00	237.40	822.60
48	2	188.04	49.36	237.40	634.56
49	3	199.32	38.07	237.40	435.24
50	4	211.28	26.11	237.40	223.96
51	5	223.96	13.44	237.40	0.00
52		1000.00	186.98	1186.98	

	A	B	C	D	E
45	Anni	Capit	Inter	Rata	Residuo
46	0				=B\$1
47	=A46+1	=D47-C47	=E46*\$B\$3	=E46*B3/((1+B3)^-B2)	=E46-B47
48	=A47+1	=D48-C48	=E47*\$B\$3	=D47	=E47-B48
49	=A48+1	=D49-C49	=E48*\$B\$3	=D48	=E48-B49
50	=A49+1	=D50-C50	=E49*\$B\$3	=D49	=E49-B50
51	=A50+1	=D51-C51	=E50*\$B\$3	=D50	=E50-B51
52		=SOMMA(B47:B51)	=SOMMA(C47:C51)	=SOMMA(D47:D51)	



- **Caso 3b: Ammortamento con quote capitali in progressione aritmetica**

$$\sum_{k=1}^n C_k = C(1 + (k-1)g)$$

$$D_0 = \sum_{k=1}^n C_k = \sum_{k=1}^n C(1 + (k-1)g) = C\left(n + \frac{n(n-1)}{2}g\right) \Rightarrow C = \frac{D_0}{n + \frac{n(n-1)}{2}g}$$

$$\sum_{k=1}^n C_k = \frac{D_0(1 + (k-1)g)}{n + \frac{n(n-1)}{2}g}$$

	A	B	C	D	E
54	<i>g</i>	20%			<i>Caparit</i>
55	<i>Anni</i>	<i>Capit</i>	<i>Inter</i>	<i>Rata</i>	<i>Residuo</i>
56	0				1000.00
57	1	142.86	60.00	202.86	857.14
58	2	171.43	51.43	222.86	685.71
59	3	200.00	41.14	241.14	485.71
60	4	228.57	29.14	257.71	257.14
61	5	257.14	15.43	272.57	0.00
62		1000.00	197.14	1197.14	

	A	B	C	D	E
54	<i>g</i>	0.2			<i>Cap arit</i>
55	<i>Anni</i>	<i>Capit</i>	<i>Inter</i>	<i>Rata</i>	<i>Residuo</i>
56	0				=B\$1
57	=A56+1	=B1/(B2+B2*(B2-1)/2*B54)	=E56*\$B\$3	=B57+C57	=E56-B57
58	=A57+1	=B\$57*(1+B\$54*(A58-1))	=E57*\$B\$3	=B58+C58	=E57-B58
59	=A58+1	=B\$57*(1+B\$54*(A59-1))	=E58*\$B\$3	=B59+C59	=E58-B59
60	=A59+1	=B\$57*(1+B\$54*(A60-1))	=E59*\$B\$3	=B60+C60	=E59-B60
61	=A60+1	=B\$57*(1+B\$54*(A61-1))	=E60*\$B\$3	=B61+C61	=E60-B61
62		=SOMMA(B57:B61)	=SOMMA(C57:C61)	=SOMMA(D57:D61)	

Nota: se $g = 0$ >>> ammortamento di tipo italiano

- Caso 3c: Ammortamento con quote capitali in progressione geometrica**

$$\sum_{k=1}^n C_k = C r_g^{k-1}, \quad D_0 = \sum_{k=1}^n C_k = C \sum_{k=1}^n r_g^{k-1} = D_0 s_{\overline{n}|g} \Rightarrow C = \frac{D_0}{s_{\overline{n}|g}}$$

$$\sum_{k=1}^n C_k = \frac{D_0 r_g^{k-1}}{s_{\overline{n}|g}} = \frac{D_0 g r_g^{k-1}}{r_g^n - 1}$$

	A	B	C	D	E
64	<i>g</i>	20%			<i>Cap geo</i>
65	<i>Anni</i>	<i>Capit</i>	<i>Inter</i>	<i>Rata</i>	<i>Residuo</i>
66	0				1000.00
67	1	134.38	60.00	194.38	865.62
68	2	161.26	51.94	213.19	704.36
69	3	193.51	42.26	235.77	510.86
70	4	232.21	30.65	262.86	278.65
71	5	278.65	16.72	295.37	0.00
72		1000.00	201.57	1201.57	

	A	B	C	D	E
64	<i>g</i>	0.2			<i>Cap geo</i>
65	<i>Anni</i>	<i>Capit</i>	<i>Inter</i>	<i>Rata</i>	<i>Residuo</i>
66	0				=B\$1
67	=A66+1	=B1*B64/((1+B64)^B2-1)	=E66*\$B\$3	=B67+C67	=E66-B67
68	=A67+1	=B\$67*(1+B\$64)^(A68-1)	=E67*\$B\$3	=B68+C68	=E67-B68
69	=A68+1	=B\$67*(1+B\$64)^(A69-1)	=E68*\$B\$3	=B69+C69	=E68-B69
70	=A69+1	=B\$67*(1+B\$64)^(A70-1)	=E69*\$B\$3	=B70+C70	=E69-B70
71	=A70+1	=B\$67*(1+B\$64)^(A71-1)	=E70*\$B\$3	=B71+C71	=E70-B71
72		=SOMMA(B67:B71)	=SOMMA(C67:C71)	=SOMMA(D67:D71)	

Nota: se $g = 0$ >>> *ammortamento di tipo italiano*

se $g \rightarrow +\infty$ >>> *ammortamento di tipo mutuo puro*

se $g = i$ >>> *ammortamento di tipo francese*

- **Caso 3d: Ammortamento con quote capitali in proporzione a numeri prefissati**

$$\sum_{k=1}^n C_k = C \beta_k, \quad D_0 = \sum_{h=1}^n C_h = C \sum_{h=1}^n \beta_h \Rightarrow C = \frac{D_0}{\sum_{h=1}^n \beta_h}$$

$$\sum_{k=1}^n C_k = D_0 \frac{\beta_k}{\sum_{h=1}^n \beta_h} = D_0 \hat{\beta}_k \quad \text{essendo} \quad \sum_{k=1}^n \hat{\beta}_k = 1$$

$$\sum_{k=1}^n D_k = D_{k-1} - D_0 \hat{\beta}_k \dots = D_0 \left(1 - \sum_{h=1}^k \hat{\beta}_h \right) = D_0 \sum_{h=k+1}^n \hat{\beta}_h$$

$$\sum_{k=1}^n I_k = i D_{k-1} \dots = i D_0 \sum_{h=k}^n \hat{\beta}_h$$

$$\sum_{k=1}^n R_k = C_k + I_k \dots = D_0 \left(\hat{\beta}_k + i \sum_{h=k}^n \hat{\beta}_h \right)$$

	A	B	C	D	E	F	Q	R
94					<i>Cap prop</i>			
95	Anni	Capit	Inter	Rata	Residuo			
96	0				1000.00			
97	1	100.00	60.00	160.00	900.00	2	10.00%	
98	2	300.00	54.00	354.00	600.00	6	30.00%	
99	3	200.00	36.00	236.00	400.00	4	20.00%	
100	4	250.00	24.00	274.00	150.00	5	25.00%	
101	5	150.00	9.00	159.00	0.00	3	15.00%	
102		1000.00	183.00	1183.00		20	100.00%	

	A	B	C	D	E
94					<i>Cap prop</i>
95	Anni	Capit	Inter	Rata	Residuo
96	0				=B\$1
97	=A96+1	=E\$96*R97	=E96*\$B\$3	=B97+C97	=E96-B97
98	=A97+1	=E\$96*R98	=E97*\$B\$3	=B98+C98	=E97-B98
99	=A98+1	=E\$96*R99	=E98*\$B\$3	=B99+C99	=E98-B99
100	=A99+1	=E\$96*R100	=E99*\$B\$3	=B100+C100	=E99-B100
101	=A100+1	=E\$96*R101	=E100*\$B\$3	=B101+C101	=E100-B101
102		=SOMMA(B97:B101)	=SOMMA(C97:C101)	=SOMMA(D97:D101)	

	Q	R
97	2	=Q97/Q\$102
98	6	=Q98/Q\$102
99	4	=Q99/Q\$102
100	5	=Q100/Q\$102
101	3	=Q101/Q\$102
102	=SOMMA(Q97:Q101)	=SOMMA(R97:R101)

Nota: >>> ammortamento di tipo italiano

$$\prod_{k=1}^n \beta_k = 1 \Rightarrow \prod_{k=1}^n \hat{\beta}_k = \frac{1}{n}$$

>>> ammortamento di tipo mutuo puro

$$\left. \begin{array}{l} \prod_{k=1}^{n-1} \beta_k = 0 \\ \beta_n = 1 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \prod_{k=1}^{n-1} \hat{\beta}_k = 0 \\ \hat{\beta}_n = 1 \end{array} \right.$$

>>> *ammortamento di tipo francese*

$$\prod_{k=1}^n \beta_k = r^{k-1} \Rightarrow \prod_{k=1}^n \hat{\beta}_k = \frac{r^{k-1}}{s_{n|i}}$$

>>> *ammortamento con quote capitali in progressione aritmetica*

$$\prod_{k=1}^n \beta_k = 1 + (k-1)g \Rightarrow \prod_{k=1}^n \hat{\beta}_k = \frac{1 + (k-1)g}{n + \frac{n(n-1)}{2}g}$$

>>> *ammortamento con quote capitali in progressione geometrica*

$$\prod_{k=1}^n \beta_k = r_g^{k-1} \Rightarrow \prod_{k=1}^n \hat{\beta}_k = \frac{r_g^{k-1}}{s_{n|g}}$$

- **Caso 5b: Ammortamento con rate in progressione aritmetica**

$$\prod_{k=1}^n R_k = \frac{D_0(1+g(k-1))}{(ia)_{n|i}^{(g)}} = \frac{D_0(1+g(k-1))i}{(1-g)(1-v^n) + g(\ddot{a}_{n|i} - nv^n)}$$

	A	B	C	D	E
74	<i>g</i>	20%			<i>Rate arit</i>
75	<i>Anni</i>	<i>Capit</i>	<i>Inter</i>	<i>Rata</i>	<i>Residuo</i>
76	0				1000.00
77	1	112.44	60.00	172.44	887.56
78	2	153.67	53.25	206.92	733.90
79	3	197.38	44.03	241.41	536.52
80	4	243.71	32.19	275.90	292.81
81	5	292.81	17.57	310.38	0.00
82		1000.00	207.05	1207.05	

	A	B	C
74	<i>g</i>	0.2	
75	<i>Anni</i>	<i>Capit</i>	<i>Inter</i>
76	0		
77	=A76+1	=D77-C77	=E76*\$B\$3
78	=A77+1	=D78-C78	=E77*\$B\$3
79	=A78+1	=D79-C79	=E78*\$B\$3
80	=A79+1	=D80-C80	=E79*\$B\$3
81	=A80+1	=D81-C81	=E80*\$B\$3
82		=SOMMA(B77:B81)	=SOMMA(C77:C81)

	D	E
74		<i>Rate arit</i>
75	<i>Rata</i>	<i>Residuo</i>
76		=B\$1
77	=B1/(((1-B74)^(1-(1+B3)^-B2)/B3+B74*(((1+B3)^(1-(1+B3)^-B2)/B3-B2*(1+B3)^-B2)/B3)	=E76-B77
78	=D\$77*(1+B\$74*(A78-1))	=E77-B78
79	=D\$77*(1+B\$74*(A79-1))	=E78-B79
80	=D\$77*(1+B\$74*(A80-1))	=E79-B80
81	=D\$77*(1+B\$74*(A81-1))	=E80-B81
82	=SOMMA(D77:D81)	

- **Caso 5c: Ammortamento con rate in progressione geometrica**

$$\prod_{k=1}^n R_k = \frac{D_0 r_g^{k-1}}{(Ga)_{\overline{n}|i}^{(g)}} = \begin{cases} \frac{D_0 r^{k-1}}{(Ga)_{\overline{n}|i}^{(i)}} = \frac{D_0 r^k}{n} & i=g \\ \frac{D_0 r_g^{k-1}}{(Ga)_{\overline{n}|i}^{(g)}} = \frac{D_0 r_g^k}{a_{\overline{n}|i}^{i-g}} & i \neq g \end{cases}$$

$$\prod_{k=1}^n D_k = \begin{cases} r D_{k-1} - \frac{D_0 r^{k-1}}{(Ga)_{\overline{n}|i}^{(g)}} = \begin{cases} r D_{k-1} - \frac{D_0 r^k}{n} & i=g \\ r D_{k-1} - \frac{D_0 r_g^k}{a_{\overline{n}|i}^{i-g}} & i \neq g \end{cases} \\ D_0 \sum_{h=k+1}^n \frac{r^{h-1} v^{h-k}}{(Ga)_{\overline{n}|i}^{(g)}} = \begin{cases} \frac{D_0 r^k}{n} \sum_{h=k+1}^n r^{h-k} v^{h-k} = D_0 r^k \frac{n-k}{n} & i=g \\ \frac{D_0 r_g^k}{(Ga)_{\overline{n}|i}^{(g)}} \sum_{h=k+1}^n r_g^{h-k-1} v^{h-k} = D_0 r_g^k \frac{(Ga)_{\overline{n-k}|i}^{(g)}}{(Ga)_{\overline{n}|i}^{(g)}} = D_0 r_g^k \frac{a_{\overline{n-k}|i}^{i-g}}{a_{\overline{n}|i}^{i-g}} & i \neq g \end{cases} \end{cases}$$

$$\prod_{k=1}^n I_k = i D_{k-1} = \begin{cases} \underbrace{i D_0 r^{k-1} \frac{n-k+1}{n}}_{i=g} \\ \underbrace{i D_0 r_g^{k-1} \frac{(Ga)_{\overline{n-k+1}|i}^{(g)}}{(Ga)_{\overline{n}|i}^{(g)}}}_{i \neq g} = i D_0 r_g^{k-1} \frac{a_{\overline{n-k+1}|i}^{i-g}}{a_{\overline{n}|i}^{i-g}} \end{cases}$$

$$\sum_{k=1}^n C_k = R_k - I_k = \begin{cases} \underbrace{\frac{D_0 r^k}{n} (1 - iv(n-k+1))}_{i=g} = \frac{D_0 r^k}{n} (1 - d(n-k+1)) \\ \underbrace{\frac{D_0 r_g^k}{(Ga)_{n/i}^{(g)}} (1 - iv_g (Ga)_{n-k+1/i}^{(g)})}_{i \neq g} = \frac{D_0 r_g^k}{a_{n/i}^{i-g}} \left(1 - iv_g a_{n-k+1/i}^{i-g} \right) \end{cases}$$

$$\sum_{k=2}^n C_k = C_{k-1} r + R_k - R_{k-1} = C_{k-1} r + R_{k-1} r_g - R_{k-1} = C_{k-1} r + R_{k-1} g$$

	A	B	C	D	E
84	<i>g</i>	20%	-11.67%		Rate geo
85	Anni	Capit	Inter	Rata	Residuo
86	0				1000.00
87	1	102.90	60.00	162.90	897.10
88	2	141.66	53.83	195.48	755.44
89	3	189.25	45.33	234.58	566.19
90	4	247.52	33.97	281.49	318.67
91	5	318.67	19.12	337.79	0.00
92		1000.00	212.24	1212.24	

	A	B	C	D	E
84	<i>g</i>	0.2	=(B3-B84)/(1+B84)		Rate geo
85	Anni	Capit	Inter	Rata	Residuo
86	0				=B\$1
87	=A86+1	=D87-C87	=E86*\$B\$3	=B1*(1+B84)*C84/(1-(1+C84)^-B2)	=E86-B87
88	=A87+1	=D88-C88	=E87*\$B\$3	=D\$87*(1+B\$84)^(A88-1)	=E87-B88
89	=A88+1	=D89-C89	=E88*\$B\$3	=D\$87*(1+B\$84)^(A89-1)	=E88-B89
90	=A89+1	=D90-C90	=E89*\$B\$3	=D\$87*(1+B\$84)^(A90-1)	=E89-B90
91	=A90+1	=D91-C91	=E90*\$B\$3	=D\$87*(1+B\$84)^(A91-1)	=E90-B91
92		=SOMMA(B87:B91)	=SOMMA(C87:C91)	=SOMMA(D87:D91)	

- **Caso 5d: Ammortamento con rate in proporzione a numeri prefissati**

$$\sum_{k=1}^n R_k = R \beta_k, \quad D_0 = \sum_{h=1}^n R_h v^h = R \sum_{h=1}^n \beta_h v^h \Rightarrow R = \frac{D_0}{\sum_{h=1}^n \beta_h v^h}$$

$$\sum_{k=1}^n R_k = D_0 \cdot \frac{\beta_k}{\sum_{h=1}^n \beta_h v^h} = D_0 \hat{\beta}_k \text{ essendo } \sum_{k=1}^n \hat{\beta}_k v^k = 1$$

$$\sum_{k=1}^n D_k = r D_{k-1} - D_0 \hat{\beta}_k \dots = D_0 \left(r^k - \sum_{h=1}^k \hat{\beta}_h r^{k-h} \right) = D_0 \sum_{h=k+1}^n \hat{\beta}_h v^{h-k}$$

$$\sum_{k=1}^n I_k = i D_{k-1} \dots = i D_0 \sum_{h=k}^n \hat{\beta}_h v^{h-k+1}$$

$$\sum_{k=1}^n C_k = R_k - I_k \dots = D_0 \left(\hat{\beta}_k - i \sum_{h=k}^n \hat{\beta}_h v^{h-k+1} \right)$$

	A	B	C	D	E	F	Q	R	S	T
104					<i>Rate prop</i>					
105	Anni	Capit	Inter	Rata	Residuo					
106	0				1000.00					
107	1	59.14	60.00	119.14	940.86	2	10.00%	9.43%	11.91%	
108	2	300.96	56.45	357.41	639.91	6	30.00%	26.70%	35.74%	
109	3	199.88	38.39	238.27	440.03	4	20.00%	16.79%	23.83%	
110	4	271.44	26.40	297.84	168.59	5	25.00%	19.80%	29.78%	
111	5	168.59	10.12	178.70	0.00	3	15.00%	11.21%	17.87%	
112		1000.00	191.36	1191.36		20	100.00%	83.94%	119.14%	

	A	B	C	D	E
104					<i>Rate prop</i>
105	Anni	Capit	Inter	Rata	Residuo
106	0				=B\$1
107	=A106+1	=D107-C107	=E106*\$B\$3	=E\$106*T107	=E106-B107
108	=A107+1	=D108-C108	=E107*\$B\$3	=E\$106*T108	=E107-B108
109	=A108+1	=D109-C109	=E108*\$B\$3	=E\$106*T109	=E108-B109
110	=A109+1	=D110-C110	=E109*\$B\$3	=E\$106*T110	=E109-B110
111	=A110+1	=D111-C111	=E110*\$B\$3	=E\$106*T111	=E110-B111
112		=SOMMA(B107:B111)	=SOMMA(C107:C111)	=SOMMA(D107:D111)	

	Q	R	S	T
107	2	=Q107/Q\$102	=R107*(1+B\$3)^-A107	=R107/S\$112
108	6	=Q108/Q\$102	=R108*(1+B\$3)^-A108	=R108/S\$112
109	4	=Q109/Q\$102	=R109*(1+B\$3)^-A109	=R109/S\$112
110	5	=Q110/Q\$102	=R110*(1+B\$3)^-A110	=R110/S\$112
111	3	=Q111/Q\$102	=R111*(1+B\$3)^-A111	=R111/S\$112
112	=SOMMA(Q107:Q111)	=SOMMA(R107:R111)	=SOMMA(S107:S111)	=SOMMA(T107:T111)

Nota: >>> ammortamento di tipo italiano

$$\prod_{k=1}^n \beta_k = 1 + (n - k + 1)i \Rightarrow \prod_{k=1}^n \hat{\beta}_k = \frac{1 + (n - k + 1)i}{n}$$

>>> ammortamento di tipo mutuo puro

$$\left. \begin{array}{l} \prod_{k=1}^{n-1} \beta_k = i \\ \beta_n = r \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \prod_{k=1}^{n-1} \hat{\beta}_k = i \\ \hat{\beta}_n = r \end{array} \right.$$

>>> ammortamento di tipo francese

$$\prod_{k=1}^n \beta_k = 1 \Rightarrow \prod_{k=1}^n \hat{\beta}_k = \frac{1}{a_{n|i}}$$

>>> ammortamento con quote capitali in progressione aritmetica

$$\prod_{k=1}^n \beta_k = 1 + (k - 1)g \Rightarrow \prod_{k=1}^n \hat{\beta}_k = \frac{1 + (k - 1)g}{(1a)_{n|i}^{(g)}}$$

>>> ammortamento con quote capitali in progressione geometrica

$$\prod_{k=1}^n \beta_k = (1+g)^{k-1} \Rightarrow \prod_{k=1}^n \hat{\beta}_k = \frac{(1+g)^{k-1}}{(Ga)_{n|i}^{(g)}} = \begin{cases} i = g \Rightarrow \frac{(1+i)^{k-1}}{(Ga)_{n|i}^{(i)}} = \frac{(1+i)^k}{n} \\ i \neq g \Rightarrow \frac{(1+g)^{k-1}}{(Ga)_{n|i}^{(g)}} = \frac{(1+g)^k}{a_{n|i-g}} \end{cases}$$

Esercizio A

Un prestito di importo P doveva essere estinto in $T (=12)$ anni secondo la metodologia dell'ammortamento francese, al tasso annuo effettivo d'interesse i .

$$D_0 = P$$

$$\prod_{t=1}^T R_t = \frac{D_0}{a_{T|i}}, \quad \prod_{t=1}^T I_t = D_{t-1} i, \quad \prod_{t=1}^T C_t = R_t - I_t, \quad \prod_{t=1}^T D_t = D_{t-1} - C_t$$

Dopo il pagamento della R -sima ($=5$) rata, il debitore ottiene la moratoria per $S (=3)$ anni, a condizione di aumentare il tasso annuo di un punto e di estinguere il debito entro la scadenza stabilita. Considerare i due seguenti casi:

- ***Caso 1: la moratoria non esonera il debitore dal pagamento delle quote interessi,***
- ***Caso 2: la moratoria esonera il debitore dal pagamento delle quote interessi.***

$$D_0 = P$$

$$\sum_{t=1}^R R_t = \frac{D_0}{a_{\overline{T}|i}}, \quad \sum_{t=R+1}^{R+S} R_t = D_{t-1} \hat{i} (sw = 1), \quad \sum_{t=R+S+1}^T R_t = \frac{D_{R+S}}{a_{\overline{T-R-S}|i}}$$

$$\sum_{t=1}^R I_t = D_{t-1} i, \quad \sum_{t=R+1}^T I_t = D_{t-1} \hat{i}$$

$$\sum_{t=1}^T C_t = R_t - I_t, \quad \sum_{t=1}^T D_t = D_{t-1} - C_t$$

	A	B	C	D	E	F	G	H
1	Capitale	1500	.	Anni	Capit	Inter	Rata	Residuo
2	Durata	12		0				1500.00
3	Durata2	4		1	99.83	60.00	159.83	1400.17
4	Tasso1	4.00%		2	103.82	56.01	159.83	1296.35
5	Tasso2	5.00%		3	107.97	51.85	159.83	1188.38
6				4	112.29	47.54	159.83	1076.08
7	Caso	1		5	116.78	43.04	159.83	959.30
8				6	121.46	38.37	159.83	837.84
9	Rata1	159.83		7	126.31	33.51	159.83	711.53
10	Rata2	270.53		8	131.37	28.46	159.83	580.16
11				9	136.62	23.21	159.83	443.54
12	Tax Val	8.00%		10	142.09	17.74	159.83	301.45
13				11	147.77	12.06	159.83	153.68
14	g	10.00%		12	153.68	6.15	159.83	0.00

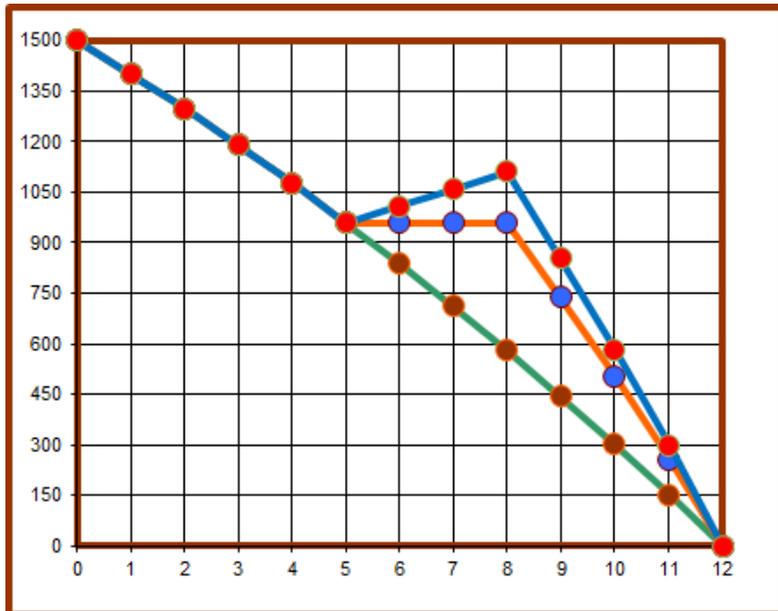
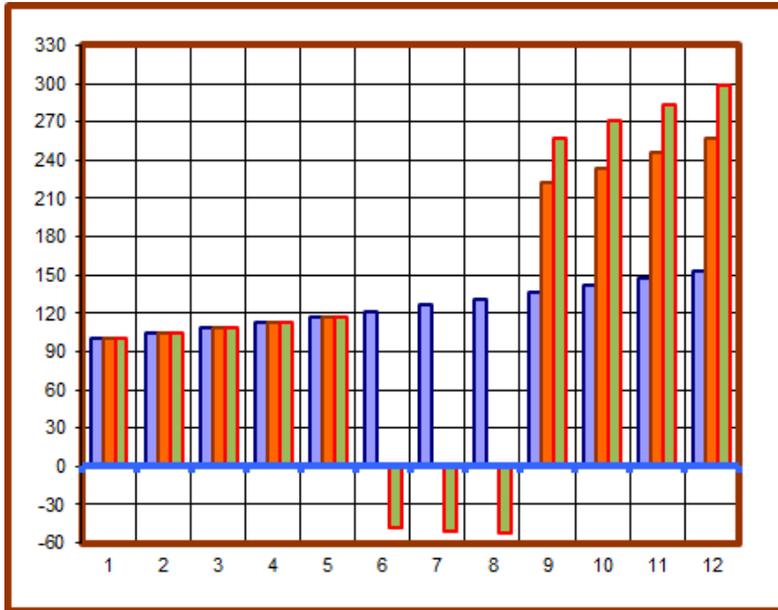
	A	B	C	I	J	K	L	M
1	<i>Capitale</i>	1500	.	<i>Anni</i>	<i>Capit</i>	<i>Inter</i>	<i>Rata</i>	<i>Residuo</i>
2	<i>Durata</i>	12		0				1500.00
3	<i>Durata2</i>	4		1	99.83	60.00	159.83	1400.17
4	<i>Tasso1</i>	4.00%		2	103.82	56.01	159.83	1296.35
5	<i>Tasso2</i>	5.00%		3	107.97	51.85	159.83	1188.38
6				4	112.29	47.54	159.83	1076.08
7	<i>Caso</i>	1		5	116.78	43.04	159.83	959.30
8				6	0.00	47.96	47.96	959.30
9	<i>Rata1</i>	159.83		7	0.00	47.96	47.96	959.30
10	<i>Rata2</i>	270.53		8	0.00	47.96	47.96	959.30
11				9	222.57	47.96	270.53	736.73
12	<i>Tax Val</i>	8.00%		10	233.70	36.84	270.53	503.03
13				11	245.38	25.15	270.53	257.65
14	<i>g</i>	10.00%		12	257.65	12.88	270.53	0.00

	A	B	C	I	J	K	L	M
1	<i>Capitale</i>	1500	.	<i>Anni</i>	<i>Capit</i>	<i>Inter</i>	<i>Rata</i>	<i>Residuo</i>
2	<i>Durata</i>	12		0				1500.00
3	<i>Durata2</i>	4		1	99.83	60.00	159.83	1400.17
4	<i>Tasso1</i>	4.00%		2	103.82	56.01	159.83	1296.35
5	<i>Tasso2</i>	5.00%		3	107.97	51.85	159.83	1188.38
6				4	112.29	47.54	159.83	1076.08
7	<i>Caso</i>	2		5	116.78	43.04	159.83	959.30
8				6	-47.96	47.96	0.00	1007.26
9	<i>Rata1</i>	159.83		7	-50.36	50.36	0.00	1057.63
10	<i>Rata2</i>	313.18		8	-52.88	52.88	0.00	1110.51
11				9	257.65	55.53	313.18	852.86
12	<i>Tax Val</i>	8.00%		10	270.53	42.64	313.18	582.32
13				11	284.06	29.12	313.18	298.26
14	<i>g</i>	10.00%		12	298.26	14.91	313.18	0.00

	A	B
1	Capitale	1500
2	Durata	12
3	Durata2	4
4	Tasso1	0.04
5	Tasso2	=B4+0.01
6		
7	Caso	2
8		
9	Rata1	=M2*B4/(1-(1+B4)^-B2)
10	Rata2	=M10*B5/(1-(1+B5)^-B3)
11		
12	Tax Val	0.08
13		
14	g	0.1

	D	E	F	G	H
1	Anni	Capit	Inter	Rata	Residuo
2	0				=B1
3	1	=G3-F3	=H2*\$B\$4	=\$B\$9	=H2-E3
4	2	=G4-F4	=H3*\$B\$4	=\$B\$9	=H3-E4
5	3	=G5-F5	=H4*\$B\$4	=\$B\$9	=H4-E5

	I	J	K	L	M
1	Anni	Capit	Inter	Rata	Residuo
2	0				=B1
3	1	=L3-K3	=M2*\$B\$4	=\$B\$9	=M2-J3
4	2	=L4-K4	=M3*\$B\$4	=\$B\$9	=M3-J4
5	3	=L5-K5	=M4*\$B\$4	=\$B\$9	=M4-J5
6	4	=L6-K6	=M5*\$B\$4	=\$B\$9	=M5-J6
7	5	=L7-K7	=M6*\$B\$4	=\$B\$9	=M6-J7
8	6	=L8-K8	=M7*\$B\$5	=K8*(\$B\$7=1)	=M7-J8
9	7	=L9-K9	=M8*\$B\$5	=K9*(\$B\$7=1)	=M8-J9
10	8	=L10-K10	=M9*\$B\$5	=K10*(\$B\$7=1)	=M9-J10
11	9	=L11-K11	=M10*\$B\$5	=\$B\$10	=M10-J11
12	10	=L12-K12	=M11*\$B\$5	=\$B\$10	=M11-J12
13	11	=L13-K13	=M12*\$B\$5	=\$B\$10	=M12-J13
14	12	=L14-K14	=M13*\$B\$5	=\$B\$10	=M13-J14



3.4 – Valutazione di un prestito

- **Tasso del prestito** i
- **Tasso del valutazione** i_1
- **Valore di un prestito (valore attuale delle rate)**

$$\prod_{h=0}^{n-1} A_h(i_1) = \sum_{k=h+1}^n R_k (1+i_1)^{-(k-h)} = \sum_{k=h+1}^n R_k v_1^{k-h}$$

$$A_0(i_1) = \sum_{k=1}^n R_k v_1^k$$

se, in particolare, il tasso di valutazione coincide con il tasso del prestito ($i_1 = i$), risulta

$$\prod_{h=0}^{n-1} A_h(i) = \sum_{k=h+1}^n R_k v^{k-h} = D_h$$

$$A_0(i) = \sum_{k=1}^n R_k v^k = D_0$$

- **Nuda proprietà (valore attuale delle quote capitale)**

$$\prod_{h=0}^{n-1} P_h(i_1) = \sum_{k=h+1}^n C_k v_1^{k-h}$$

$$P_0(i_1) = \sum_{k=1}^n C_k v_1^k$$

- **Usufrutto (valore attuale delle quote interessi)**

$$\prod_{h=0}^{n-1} U_h(i_1) = \sum_{k=h+1}^n I_k v_1^{k-h}$$

$$U_0(i_1) = \sum_{k=1}^n I_k v_1^k$$

- **Indice del grado di capitalizzazione (rapporto tra l'usufrutto e il valore del prestito)**

$$\prod_{h=0}^{n-1} G_h(i_1) = \frac{U_h(i_1)}{A_h(i_1)} = \frac{\sum_{k=h+1}^n I_k v_1^{k-h}}{\sum_{k=h+1}^n R_k v_1^{k-h}}$$

$$G_0(i_1) = \frac{U_0(i_1)}{A_0(i_1)} = \frac{\sum_{k=1}^n I_k v_1^k}{\sum_{k=1}^n R_k v_1^k}$$

se, in particolare, il tasso di valutazione coincide con il tasso del prestito ($i_1 = i$), risulta

$$\prod_{h=0}^{n-1} G_h(i) = \frac{U_h(i)}{D_h} = \frac{\sum_{k=h+1}^n I_k v^{k-h}}{\sum_{k=h+1}^n R_k v^{k-h}}$$

$$G_0(i) = \frac{U_0(i)}{D_0} = \frac{\sum_{k=1}^n I_k v^k}{\sum_{k=1}^n R_k v^k}$$

da cui possono trarsi diverse relazioni ricorrenti:

$$\prod_{h=1}^{n-1} A_{h-1}(i_1) = (A_h(i_1) + R_h) v_1, \quad A_{n-1}(i_1) = R_n v_1$$

$$\prod_{h=1}^{n-1} P_{h-1}(i_1) = (P_h(i_1) + C_h) v_1, \quad P_{n-1}(i_1) = C_n v_1$$

$$\prod_{h=1}^{n-1} U_{h-1}(i_1) = (U_h(i_1) + I_h) v_1, \quad U_{n-1}(i_1) = I_n v_1$$

$$\prod_{h=1}^{n-1} G_{h-1}(i_1) = \frac{U_{h-1}(i_1)}{A_{h-1}(i_1)} = \frac{U_h(i_1) + I_h}{A_h(i_1) + R_h} = \frac{G_h(i_1) A_h(i_1) + I_h}{A_h(i_1) + R_h}$$

$$G_{n-1}(i_1) = \frac{U_{n-1}(i_1)}{A_{n-1}(i_1)} = \frac{I_n}{R_n}$$

• **Formula di Makeham**

$$\begin{aligned}
 \prod_{h=0}^{n-1} A_h(i_1) &= \sum_{k=h+1}^n R_k v_1^{k-h} = \sum_{k=h+1}^n (C_k + I_k) v_1^{k-h} = P_h(i_1) + i \sum_{k=h+1}^n D_{k-1} v_1^{k-h} = \\
 &= P_h(i_1) + i \sum_{k=h+1}^n \sum_{g=k}^n C_g v_1^{k-h} = P_h(i_1) + i \sum_{g=h+1}^n C_g \sum_{k=h+1}^g v_1^{k-h} = P_h(i_1) + i \sum_{g=h+1}^n C_g a_{\overline{g-h}|i_1} = \\
 &= P_h(i_1) + \frac{i}{i_1} \sum_{g=h+1}^n C_g (1 - v_1^{g-h}) = P_h(i_1) + \frac{i}{i_1} \left(\sum_{g=h+1}^n C_g - \sum_{g=h+1}^n C_g v_1^{g-h} \right) = \\
 &= P_h(i_1) + \underbrace{\frac{i}{i_1} (D_h - P_h(i_1))}_{U_h(i_1)} = \underbrace{D_h}_{A_h(i)} + \underbrace{\frac{i-i_1}{i_1} (D_h - P_h(i_1))}_{A_h(i_1) - A_h(i)}
 \end{aligned}$$

$$A_0(i_1) = P_0(i_1) + \frac{i}{i_1} (D_0 - P_0(i_1)) = D_0 + \frac{i-i_1}{i_1} (D_0 - P_0(i_1))$$

$U_0(i_1)$
 $A_0(i)$
 $A_0(i_1) - A_0(i)$

$$\begin{aligned}
 \prod_{h=0}^{n-1} A_h(i_1) &= P_h(i_1) + \frac{i}{i_1} (D_h - P_h(i_1)) \Rightarrow (A_h(i_1) - P_h(i_1)) i_1 = (D_h - P_h(i_1)) i \Rightarrow \\
 &\Rightarrow P_h(i_1) (i - i_1) = (D_h i - A_h(i_1) i_1)
 \end{aligned}$$

$$\prod_{h=0}^{n-1} P_h(i_1) = \frac{D_h i - A_h(i_1) i_1}{i - i_1}$$

$$P_0(i_1) = \frac{D_0 i - A_0(i_1) i_1}{i - i_1}$$

$$\prod_{h=0}^{n-1} U_h(i_1) = \frac{(A_h(i_1) - D_h) i}{i - i_1}$$

$$U_0(i_1) = \frac{(A_0(i_1) - D_0) i}{i - i_1}$$

$$\prod_{h=0}^{n-1} G_h(i_1) = \frac{\left(1 - \frac{D_h}{A_h(i_1)}\right) i}{i - i_1}$$

$$G_0(i_1) = \frac{\left(1 - \frac{D_0}{A_0(i_1)}\right) i}{i - i_1}$$

Esercizio (da esercizio precedente)

Calcolare la nuda proprietà, l'usufrutto, il valore del prestito e l'indice del grado di capitalizzazione, relativamente ad un tasso di valutazione i_1 (es. 5%) nei diversi casi sotto indicati:

- **Caso 1: Ammortamento con quote capitali prefissate**

	G	H	I	J	K	L	M	N
5	5%	Nprop	Usuf	Valore	Nprop	Usuf	Valore	Grado
6	1.0000				862.20	165.36	1027.56	16.09%
7	0.9524	285.71	57.14	342.86	605.31	113.62	718.94	15.80%
8	0.9070	90.70	38.10	128.80	535.58	77.30	612.88	12.61%
9	0.8638	0.00	31.10	31.10	562.36	45.17	607.53	7.44%
10	0.8227	329.08	29.62	358.70	190.48	11.43	201.90	5.66%
11	0.7835	156.71	9.40	166.11				
12		862.20	165.36	1027.56				

	G	H	I	J
5	0.05	Nprop	Usuf	Valore
6	$=(1+G\$5)^{-A6}$			
7	$=(1+G\$5)^{-A7}$	$=B7*\$G7$	$=C7*\$G7$	$=D7*\$G7$
8	$=(1+G\$5)^{-A8}$	$=B8*\$G8$	$=C8*\$G8$	$=D8*\$G8$
9	$=(1+G\$5)^{-A9}$	$=B9*\$G9$	$=C9*\$G9$	$=D9*\$G9$
10	$=(1+G\$5)^{-A10}$	$=B10*\$G10$	$=C10*\$G10$	$=D10*\$G10$
11	$=(1+G\$5)^{-A11}$	$=B11*\$G11$	$=C11*\$G11$	$=D11*\$G11$
12		$=SOMMA(H7:H11)$	$=SOMMA(I7:I11)$	$=SOMMA(J7:J11)$

	K	L
5	Nprop	Usuf
6	$=(K7+B7)*\$G7/\$G6$	$=(L7+C7)*\$G7/\$G6$
7	$=(K8+B8)*\$G8/\$G7$	$=(L8+C8)*\$G8/\$G7$
8	$=(K9+B9)*\$G9/\$G8$	$=(L9+C9)*\$G9/\$G8$
9	$=(K10+B10)*\$G10/\$G9$	$=(L10+C10)*\$G10/\$G9$
10	$=(K11+B11)*\$G11/\$G10$	$=(L11+C11)*\$G11/\$G10$

	M	N
5	Valore	Grado
6	$=(M7+D7)*\$G7/\$G6$	$=L6/M6$
7	$=(M8+D8)*\$G8/\$G7$	$=L7/M7$
8	$=(M9+D9)*\$G9/\$G8$	$=L8/M8$
9	$=(M10+D10)*\$G10/\$G9$	$=L9/M9$
10	$=(M11+D11)*\$G11/\$G10$	$=L10/M10$

- **Caso 2: Ammortamento di mutuo puro**

$$\prod_{h=0}^{n-1} P_h(i_1) = D_0 v_1^{n-h}$$

$$\prod_{h=0}^{n-1} U_h(i_1) = D_0 i a_{\overline{n-h}|i_1} = D_0 \frac{i}{i_1} (1 - v_1^{n-h})$$

$$\begin{aligned} \prod_{h=0}^{n-1} A_h(i_1) &= P_h(i_1) + U_h(i_1) = D_0 v_1^{n-h} + D_0 i a_{\overline{n-h}|i_1} = D_0 v_1^{n-h} + D_0 i \frac{1 - v_1^{n-h}}{i_1} = \\ &= \frac{D_0}{i_1} (i_1 v_1^{n-h} + i(1 - v_1^{n-h})) = \frac{D_0}{i_1} (i + (i_1 - i)v_1^{n-h}) \end{aligned}$$

$$\prod_{h=0}^{n-1} G_h(i_1) = \frac{U_h(i_1)}{A_h(i_1)} = \frac{D_0 \frac{i}{i_1} (1 - v_1^{n-h})}{\frac{D_0}{i_1} (i + (i_1 - i)v_1^{n-h})} = \frac{i(1 - v_1^{n-h})}{i + (i_1 - i)v_1^{n-h}}$$

se, in particolare, il tasso di valutazione coincide con il tasso del prestito ($i_1 = i$), risulta

$$\prod_{h=0}^{n-1} A_h(i) = D_h = D_0$$

$$\prod_{h=0}^{n-1} P_h(i) = D_0 v^{n-h}$$

$$\prod_{h=0}^{n-1} U_h(i) = D_0 (1 - v^{n-h})$$

$$\prod_{h=0}^{n-1} G_h(i_1) = 1 - v^{n-h}$$

	G	H	I	J	K	L	M	N
15		N.Pr.	Usuf	Valore	Nprop	Usuf	Valore	Grado
16	1.0000				783.53	259.77	1043.29	24.90%
17	0.9524	0.00	57.14	57.14	822.70	212.76	1035.46	20.55%
18	0.9070	0.00	54.42	54.42	863.84	163.39	1027.23	15.91%
19	0.8638	0.00	51.83	51.83	907.03	111.56	1018.59	10.95%
20	0.8227	0.00	49.36	49.36	952.38	57.14	1009.52	5.66%
21	0.7835	783.53	47.01	830.54				
22		783.53	259.77	1043.29				

	G	H	I	J
15		N.Pr.	Usuf	Valore
16	$=(1+G\$5)^{-A16}$			
17	$=(1+G\$5)^{-A17}$	$=B17*\$G17$	$=C17*\$G17$	$=D17*\$G17$
18	$=(1+G\$5)^{-A18}$	$=B18*\$G18$	$=C18*\$G18$	$=D18*\$G18$
19	$=(1+G\$5)^{-A19}$	$=B19*\$G19$	$=C19*\$G19$	$=D19*\$G19$
20	$=(1+G\$5)^{-A20}$	$=B20*\$G20$	$=C20*\$G20$	$=D20*\$G20$
21	$=(1+G\$5)^{-A21}$	$=B21*\$G21$	$=C21*\$G21$	$=D21*\$G21$
22		$=SOMMA(H17:H21)$	$=SOMMA(I17:I21)$	$=SOMMA(J17:J21)$

	K	L
15	Nprop	Usuf
16	$=(K17+B17)*\$G17/\$G16$	$=(L17+C17)*\$G17/\$G16$
17	$=(K18+B18)*\$G18/\$G17$	$=(L18+C18)*\$G18/\$G17$
18	$=(K19+B19)*\$G19/\$G18$	$=(L19+C19)*\$G19/\$G18$
19	$=(K20+B20)*\$G20/\$G19$	$=(L20+C20)*\$G20/\$G19$
20	$=(K21+B21)*\$G21/\$G20$	$=(L21+C21)*\$G21/\$G20$

	M	N
15	Valore	Grado
16	$=(M17+D17)*\$G17/\$G16$	$=L16/M16$
17	$=(M18+D18)*\$G18/\$G17$	$=L17/M17$
18	$=(M19+D19)*\$G19/\$G18$	$=L18/M18$
19	$=(M20+D20)*\$G20/\$G19$	$=L19/M19$
20	$=(M21+D21)*\$G21/\$G20$	$=L20/M20$

- **Caso 3: Ammortamento di tipo italiano**

$$\sum_{h=0}^{n-1} P_h(i_1) = \frac{D_0}{n} a_{\overline{n-h}|i_1}$$

$$\sum_{h=0}^{n-1} U_h(i_1) = \frac{i}{i_1} (D_h - P_h(i_1)) = \frac{i}{i_1} \left(D_0 \frac{n-h}{n} - \frac{D_0}{n} a_{\overline{n-h}|i_1} \right) = \frac{D_0}{n} \frac{i}{i_1} (n-h - a_{\overline{n-h}|i_1})$$

$$\begin{aligned} \sum_{h=0}^{n-1} A_h(i_1) &= P_h(i_1) + U_h(i_1) = \frac{D_0}{n} a_{\overline{n-h}|i_1} + \frac{D_0}{n} \frac{i}{i_1} (n-h - a_{\overline{n-h}|i_1}) = \\ &= \frac{D_0}{n} \left(a_{\overline{n-h}|i_1} + \frac{i}{i_1} (n-h - a_{\overline{n-h}|i_1}) \right) \end{aligned}$$

$$\sum_{h=0}^{n-1} G_h(i_1) = \frac{U_h(i_1)}{A_h(i_1)} = \frac{\frac{D_0}{n} \frac{i}{i_1} (n-h - a_{\overline{n-h}|i_1})}{\frac{D_0}{n} \left(a_{\overline{n-h}|i_1} + \frac{i}{i_1} (n-h - a_{\overline{n-h}|i_1}) \right)} = \frac{n-h - a_{\overline{n-h}|i_1}}{a_{\overline{n-h}|i_1} + \frac{i}{i_1} (n-h - a_{\overline{n-h}|i_1})}$$

se, in particolare, il tasso di valutazione coincide con il tasso del prestito ($i_1 = i$), risulta

$$\sum_{h=0}^{n-1} A_h(i) = D_h = \frac{D_0}{n} (n-h)$$

$$\sum_{h=0}^{n-1} P_h(i) = \frac{D_0}{n} a_{\overline{n-h}|i}$$

$$\sum_{h=0}^{n-1} U_h(i) = \frac{D_0}{n} (n-h - a_{\overline{n-h}|i})$$

$$\sum_{h=0}^{n-1} G_h(i) = 1 - \frac{a_{\overline{n-h}|i}}{n-h}$$

	G	H	I	J	K	L	M	N
25		N.Pr.	Usuf	Valore	Nprop	Usuf	Valore	Grado
26	1.0000				865.90	160.93	1026.82	15.67%
27	0.9524	190.48	57.14	247.62	709.19	108.97	818.16	13.32%
28	0.9070	181.41	43.54	224.94	544.65	66.42	611.07	10.87%
29	0.8638	172.77	31.10	203.87	371.88	33.74	405.62	8.32%
30	0.8227	164.54	19.74	184.29	190.48	11.43	201.90	5.66%
31	0.7835	156.71	9.40	166.11				
32		865.90	160.93	1026.82				

	G	H	I	J
25		N.Pr.	Usuf	Valore
26	$=(1+G\$5)^{-A26}$			
27	$=(1+G\$5)^{-A27}$	$=B27*\$G27$	$=C27*\$G27$	$=D27*\$G27$
28	$=(1+G\$5)^{-A28}$	$=B28*\$G28$	$=C28*\$G28$	$=D28*\$G28$
29	$=(1+G\$5)^{-A29}$	$=B29*\$G29$	$=C29*\$G29$	$=D29*\$G29$
30	$=(1+G\$5)^{-A30}$	$=B30*\$G30$	$=C30*\$G30$	$=D30*\$G30$
31	$=(1+G\$5)^{-A31}$	$=B31*\$G31$	$=C31*\$G31$	$=D31*\$G31$
32		$=SOMMA(H27:H31)$	$=SOMMA(I27:I31)$	$=SOMMA(J27:J31)$

	K	L
25	Nprop	Usuf
26	$=(K27+B27)*\$G27/\$G26$	$=(L27+C27)*\$G27/\$G26$
27	$=(K28+B28)*\$G28/\$G27$	$=(L28+C28)*\$G28/\$G27$
28	$=(K29+B29)*\$G29/\$G28$	$=(L29+C29)*\$G29/\$G28$
29	$=(K30+B30)*\$G30/\$G29$	$=(L30+C30)*\$G30/\$G29$
30	$=(K31+B31)*\$G31/\$G30$	$=(L31+C31)*\$G31/\$G30$

	M	N
25	Valore	Grado
26	$=(M27+D27)*\$G27/\$G26$	$=L26/M26$
27	$=(M28+D28)*\$G28/\$G27$	$=L27/M27$
28	$=(M29+D29)*\$G29/\$G28$	$=L28/M28$
29	$=(M30+D30)*\$G30/\$G29$	$=L29/M29$
30	$=(M31+D31)*\$G31/\$G30$	$=L30/M30$

- **Caso 4: Ammortamento con rate prefissate**

	G	H	I	J	K	L	M	N
35		N.Pr.	Usuf	Valore	Nprop	Usuf	Valore	Grado
36	1.0000				842.55	188.94	1031.49	18.32%
37	0.9524	180.95	57.14	238.10	694.67	138.39	833.07	16.61%
38	0.9070	119.18	44.08	163.27	598.01	96.71	694.72	13.92%
39	0.8638	-35.17	35.17	0.00	668.62	60.83	729.45	8.34%
40	0.8227	293.57	35.51	329.08	345.21	20.71	365.93	5.66%
41	0.7835	284.01	17.04	301.05				
42		842.55	188.94	1031.49				

	G	H	I	J
35		N.Pr.	Usuf	Valore
36	$=(1+G\$5)^{-A36}$			
37	$=(1+G\$5)^{-A37}$	$=B37*\$G37$	$=C37*\$G37$	$=D37*\$G37$
38	$=(1+G\$5)^{-A38}$	$=B38*\$G38$	$=C38*\$G38$	$=D38*\$G38$
39	$=(1+G\$5)^{-A39}$	$=B39*\$G39$	$=C39*\$G39$	$=D39*\$G39$
40	$=(1+G\$5)^{-A40}$	$=B40*\$G40$	$=C40*\$G40$	$=D40*\$G40$
41	$=(1+G\$5)^{-A41}$	$=B41*\$G41$	$=C41*\$G41$	$=D41*\$G41$
42		$=SOMMA(H37:H41)$	$=SOMMA(I37:I41)$	$=SOMMA(J37:J41)$

	K	L
35	Nprop	Usuf
36	$=(K37+B37)*\$G37/\$G36$	$=(L37+C37)*\$G37/\$G36$
37	$=(K38+B38)*\$G38/\$G37$	$=(L38+C38)*\$G38/\$G37$
38	$=(K39+B39)*\$G39/\$G38$	$=(L39+C39)*\$G39/\$G38$
39	$=(K40+B40)*\$G40/\$G39$	$=(L40+C40)*\$G40/\$G39$
40	$=(K41+B41)*\$G41/\$G40$	$=(L41+C41)*\$G41/\$G40$

	M	N
35	Valore	Grado
36	$=(M37+D37)*\$G37/\$G36$	$=L36/M36$
37	$=(M38+D38)*\$G38/\$G37$	$=L37/M37$
38	$=(M39+D39)*\$G39/\$G38$	$=L38/M38$
39	$=(M40+D40)*\$G40/\$G39$	$=L39/M39$
40	$=(M41+D41)*\$G41/\$G40$	$=L40/M40$

- **Caso 5: Ammortamento di tipo francese**

$$\sum_{h=0}^{n-1} A_h(i_1) = D_0 \frac{a_{\overline{n-h}|i_1}}{a_{\overline{n}|i}}$$

$$\sum_{h=0}^{n-1} P_h(i_1) = \frac{D_h i - A_h(i_1) i_1}{i - i_1} = \frac{D_0 \frac{a_{\overline{n-h}|i}}{a_{\overline{n}|i}} i - D_0 \frac{a_{\overline{n-h}|i_1}}{a_{\overline{n}|i}} i_1}{i - i_1} = \frac{D_0 v_1^{n-h} - v^{n-h}}{a_{\overline{n}|i} (i - i_1)} =$$

$$\sum_{h=0}^{n-1} U_h(i_1) = \frac{(A_h(i_1) - D_h) i}{i - i_1} = \frac{\left(D_0 \frac{a_{\overline{n-h}|i_1}}{a_{\overline{n}|i}} - D_0 \frac{a_{\overline{n-h}|i}}{a_{\overline{n}|i}} \right) i}{i - i_1} = \frac{D_0 i \frac{a_{\overline{n-h}|i_1} - a_{\overline{n-h}|i}}{a_{\overline{n}|i}}}{i - i_1}$$

$$\sum_{h=0}^{n-1} G_h(i_1) = \frac{U_h(i_1)}{A_h(i_1)} = \frac{\frac{D_0 i \frac{a_{\overline{n-h}|i_1} - a_{\overline{n-h}|i}}{a_{\overline{n}|i}}}{i - i_1}}{D_0 \frac{a_{\overline{n-h}|i_1}}{a_{\overline{n}|i}}} = \frac{i \frac{a_{\overline{n-h}|i_1} - a_{\overline{n-h}|i}}{a_{\overline{n-h}|i_1}}}{i - i_1}$$

se, in particolare, il tasso di valutazione coincide con il tasso del prestito ($i_1 = i$), risulta

$$\sum_{h=0}^{n-1} A_h(i) = D_h = D_0 \frac{a_{\overline{n-h}|i}}{a_{\overline{n}|i}}$$

$$\sum_{h=0}^{n-1} P_h(i) = \frac{D_0}{s_{\overline{n}|i}} (Ga)_{\overline{n-h}|i}^{(i)} = \frac{D_0}{a_{\overline{n}|i}} (n-h)v^{n-h+1}$$

$$\sum_{h=0}^{n-1} U_h(i) = D_0 \frac{a_{\overline{n-h}|i}}{a_{\overline{n}|i}} - \frac{D_0}{a_{\overline{n}|i}} (n-h)v^{n-h+1} = \frac{D_0}{a_{\overline{n}|i}} (a_{\overline{n-h}|i} - (n-h)v^{n-h+1})$$

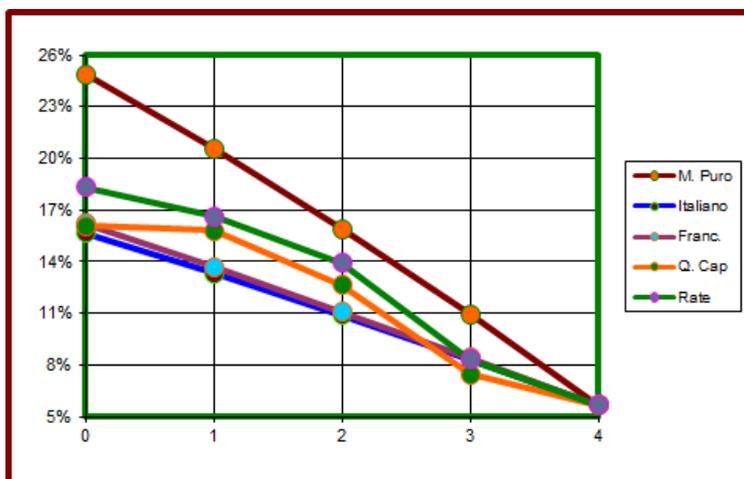
$$\sum_{h=0}^{n-1} G_h(i) = 1 - \frac{(n-h)v^{n-h+1}}{a_{\overline{n-h}|i}} = 1 - \frac{n-h}{\ddot{s}_{\overline{n-h}|i}}$$

	G	H	I	J	K	L	M	N
45		N.Pr.	Usuf	Valore				Grado
46	1.0000				860.99	166.81	1027.80	16.23%
47	0.9524	168.95	57.14	226.09	726.64	115.15	841.80	13.68%
48	0.9070	170.56	44.77	215.33	574.93	71.56	646.49	11.07%
49	0.8638	172.18	32.89	205.07	404.36	37.06	441.42	8.40%
50	0.8227	173.82	21.48	195.31	213.29	12.80	226.09	5.66%
51	0.7835	175.48	10.53	186.01				
52		860.99	166.81	1027.80				

	G	H	I	J
45		N.Pr.	Usuf	Valore
46	$=(1+G\$5)^{-A46}$			
47	$=(1+G\$5)^{-A47}$	$=B47*\$G47$	$=C47*\$G47$	$=D47*\$G47$
48	$=(1+G\$5)^{-A48}$	$=B48*\$G48$	$=C48*\$G48$	$=D48*\$G48$
49	$=(1+G\$5)^{-A49}$	$=B49*\$G49$	$=C49*\$G49$	$=D49*\$G49$
50	$=(1+G\$5)^{-A50}$	$=B50*\$G50$	$=C50*\$G50$	$=D50*\$G50$
51	$=(1+G\$5)^{-A51}$	$=B51*\$G51$	$=C51*\$G51$	$=D51*\$G51$
52		$=SOMMA(H47:H51)$	$=SOMMA(I47:I51)$	$=SOMMA(J47:J51)$

	K	L
45		
46	$=(K47+B47)*\$G47/\$G46$	$=(L47+C47)*\$G47/\$G46$
47	$=(K48+B48)*\$G48/\$G47$	$=(L48+C48)*\$G48/\$G47$
48	$=(K49+B49)*\$G49/\$G48$	$=(L49+C49)*\$G49/\$G48$
49	$=(K50+B50)*\$G50/\$G49$	$=(L50+C50)*\$G50/\$G49$
50	$=(K51+B51)*\$G51/\$G50$	$=(L51+C51)*\$G51/\$G50$

	M	N
45		Grado
46	$=(M47+D47)*\$G47/\$G46$	$=L46/M46$
47	$=(M48+D48)*\$G48/\$G47$	$=L47/M47$
48	$=(M49+D49)*\$G49/\$G48$	$=L48/M48$
49	$=(M50+D50)*\$G50/\$G49$	$=L49/M49$
50	$=(M51+D51)*\$G51/\$G50$	$=L50/M50$



- **Caso 3b: Ammortamento con quote capitali in progressione aritmetica (controllare formule sul foglio excel)**

	G	H	I	J	K	L	M	N
55		N.Pr.	Usuf	Valore				Grado
56	1.0000				853.84	175.40	1029.23	17.04%
57	0.9524	136.05	57.14	193.20	753.67	124.17	877.84	14.14%
58	0.9070	155.49	46.65	202.14	619.93	78.94	698.87	11.30%
59	0.8638	172.77	35.54	208.31	450.92	41.75	492.67	8.47%
60	0.8227	188.05	23.98	212.02	244.90	14.69	259.59	5.66%
61	0.7835	201.48	12.09	213.57				
62		853.84	175.40	1029.23				

- **Caso 3c: Ammortamento con quote capitali in progressione geometrica (controllare formule sul foglio excel)**

	G	H	I	J	K	L	M	N
65		N.Pr.	Usuf	Valore				Grado
66	1.0000				860.99	166.81	1027.80	16.23%
67	0.9524	168.95	57.14	226.09	726.64	115.15	841.80	13.68%
68	0.9070	170.56	44.77	215.33	574.93	71.56	646.49	11.07%
69	0.8638	172.18	32.89	205.07	404.36	37.06	441.42	8.40%
70	0.8227	173.82	21.48	195.31	213.29	12.80	226.09	5.66%
71	0.7835	175.48	10.53	186.01				
72		860.99	166.81	1027.80				

- **Caso 3d: Ammortamento con quote capitali in proporzione a numeri prefissati (controllare formule sul foglio excel)**

	G	H	I	J	K	L	M	N
95		Nprop	Usuf	Valore	Nprop	Usuf	Valore	Grado
96	1.0000				863.32	164.02	1027.34	15.97%
97	0.9524	95.24	57.14	152.38	806.48	112.22	918.70	12.21%
98	0.9070	272.11	48.98	321.09	546.81	63.83	610.64	10.45%
99	0.8638	172.77	31.10	203.87	374.15	31.02	405.17	7.66%
100	0.8227	205.68	19.74	225.42	142.86	8.57	151.43	5.66%
101	0.7835	117.53	7.05	124.58				
102		863.32	164.02	1027.34				

- **Caso 5b: Ammortamento con rate in progressione aritmetica (controllare formule sul foglio excel)**

	G	H	I	J	K	L	M	N
75		N.Pr.	Usuf	Valore				Grado
76	1.0000				846.89	183.73	1030.62	17.83%
77	0.9524	107.08	57.14	164.22	776.80	132.92	909.72	14.61%
78	0.9070	139.38	48.30	187.68	661.97	86.31	748.28	11.53%
79	0.8638	170.50	38.04	208.54	497.69	46.59	544.29	8.56%
80	0.8227	200.50	26.48	226.98	278.87	16.73	295.60	5.66%
81	0.7835	229.43	13.77	243.19				
82		846.89	183.73	1030.62				

- **Caso 5c: Ammortamento con rate in progressione geometrica (controllare formule sul foglio excel)**

	G	H	I	J	K	L	M	N
85		N.Pr.	Usuf	Valore				Grado
86	1.0000				843.29	188.05	1031.34	18.23%
87	0.9524	98.00	57.14	155.14	782.56	137.45	920.01	14.94%
88	0.9070	128.49	48.82	177.31	680.03	90.50	770.53	11.74%
89	0.8638	163.48	39.15	202.64	524.78	49.70	574.48	8.65%
90	0.8227	203.64	27.95	231.58	303.50	18.21	321.71	5.66%
91	0.7835	249.69	14.98	264.67				
92		843.29	188.05	1031.34				

- **Caso 5d: Ammortamento con rate in proporzione a numeri prefissati (controllare formule sul foglio excel)**

	G	H	I	J	K	L	M	N
105		N.Pr.	Usuf	Valore	Nprop	Usuf	Valore	Grado
106	1.0000				857.37	171.16	1028.53	16.64%
107	0.9524	56.32	57.14	113.46	841.10	119.72	960.82	12.46%
108	0.9070	272.98	51.20	324.18	582.20	69.25	651.45	10.63%
109	0.8638	172.66	33.17	205.83	411.43	34.32	445.75	7.70%
110	0.8227	223.31	21.72	245.03	160.56	9.63	170.19	5.66%
111	0.7835	132.09	7.93	140.02				
112		857.37	171.16	1028.53				

Esercizio A (proseguimento)

Calcolare la nuda proprietà, l'usufrutto, il valore del prestito e l'indice del grado di capitalizzazione, relativamente ad un tasso di valutazione i_t (es. 8%)

	A	B	C	N	O	P	Q
1	Capitale	1500	.	Nprop	Usuf	Valore	Grado
2	Durata	12		856.32	350.06	1206.38	29.02%
3	Durata2	4		825.00	318.06	1143.06	27.83%
4	Tasso1	4.00%		787.18	287.50	1074.68	26.75%
5	Tasso2	5.00%		742.18	258.65	1000.82	25.84%
6				689.26	231.80	921.06	25.17%
7	Caso	1		627.61	207.30	834.92	24.83%
8				677.82	175.92	853.74	20.61%
9	Rata1	159.83		732.05	142.03	874.08	16.25%
10	Rata2	270.53		790.61	105.43	896.04	11.77%
11				631.29	65.90	697.19	9.45%
12	Tax Val	8.00%		448.10	34.33	482.43	7.12%
13				238.57	11.93	250.49	4.76%
14	g	10.00%					

	A	B	C	N	O	P	Q
1	Capitale	1500	.	Nprop	Usuf	Valore	Grado
2	Durata	12		835.47	363.09	1198.56	30.29%
3	Durata2	4		802.48	332.14	1134.61	29.27%
4	Tasso1	4.00%		762.85	302.70	1065.55	28.41%
5	Tasso2	5.00%		715.91	275.06	990.97	27.76%
6				660.89	249.53	910.42	27.41%
7	Caso	2		596.97	226.45	823.43	27.50%
8				692.69	196.61	889.30	22.11%
9	Rata1	159.83		798.47	161.97	960.44	16.86%
10	Rata2	313.18		915.23	122.05	1037.28	11.77%
11				730.80	76.29	807.09	9.45%
12	Tax Val	8.00%		518.73	39.75	558.48	7.12%
13				276.17	13.81	289.98	4.76%
14	g	10.00%					

	N	O	P	Q
1	Nprop	Usuf	Valore	Grado
2	$= (N3+J3)/(1+SB\$12)$	$= (O3+K3)/(1+SB\$12)$	$= (P3+L3)/(1+SB\$12)$	$= O2/P2$
3	$= (N4+J4)/(1+SB\$12)$	$= (O4+K4)/(1+SB\$12)$	$= (P4+L4)/(1+SB\$12)$	$= O3/P3$
4	$= (N5+J5)/(1+SB\$12)$	$= (O5+K5)/(1+SB\$12)$	$= (P5+L5)/(1+SB\$12)$	$= O4/P4$

Esercizio A (proseguimento)

Ripetizione dell'esercizio nell'ipotesi che le rate costituiscano una progressione aritmetica di ragione relativa g ($=10\%$)

$$D_0 = P$$

$$\prod_{t=1}^T R_t = \frac{D_0(1+g(t-1))}{(1a)_{\overline{T}|i}^{(g)}}, \quad \prod_{t=1}^T I_t = D_{t-1}i, \quad \prod_{t=1}^T C_t = R_t - I_t, \quad \prod_{t=1}^T D_t = D_{t-1} - C_t$$

$$D_0 = P$$

$$\prod_{t=1}^R R_t = \frac{D_0(1+g(t-1))}{(1a)_{\overline{T}|i}^{(g)}}, \quad \prod_{t=R+1}^{R+S} R_t = D_{t-1}\hat{i} \quad (sw = 1)$$

$$\prod_{t=R+S+1}^T R_t = \frac{D_{R+S}(1+g(t-R-S-1))}{(1a)_{\overline{T-R-S}|i}^{(g)}}$$

$$\prod_{t=1}^R I_t = D_{t-1}i, \quad \prod_{t=R+1}^T I_t = D_{t-1}\hat{i}$$

$$\prod_{t=1}^T C_t = R_t - I_t, \quad \prod_{t=1}^T D_t = D_{t-1} - C_t$$

	A	B	C	D	E	F	G	H
16	<i>Capitale</i>	1500		<i>Anni</i>	<i>Capit</i>	<i>Inter</i>	<i>Rata</i>	<i>Residuo</i>
17	<i>Durata</i>	12		0				1500.00
18	<i>Durata2</i>	4		1	46.31	60.00	106.31	1453.69
19	<i>Tasso1</i>	4.00%		2	58.79	58.15	116.94	1394.90
20	<i>Tasso2</i>	5.00%		3	71.77	55.80	127.57	1323.12
21				4	85.28	52.92	138.20	1237.85
22	<i>Caso</i>	1		5	99.32	49.51	148.83	1138.53
23		9.3851		6	113.92	45.54	159.46	1024.61
24	<i>Rata1</i>	106.31		7	129.11	40.98	170.09	895.50
25	<i>Rata2</i>	280.69		8	144.90	35.82	180.72	750.59
26		3.5460		9	161.33	30.02	191.36	589.26
27	<i>Tax Val</i>	8.00%		10	178.42	23.57	201.99	410.85
28				11	196.18	16.43	212.62	214.66
29	<i>g</i>	10.00%		12	214.66	8.59	223.25	0.00

	A	B	C	I	J	K	L	M
16	<i>Capitale</i>	1500		<i>Anni</i>	<i>Capit</i>	<i>Inter</i>	<i>Rata</i>	<i>Residuo</i>
17	<i>Durata</i>	12		0				1500.00
18	<i>Durata2</i>	4		1	46.31	60.00	106.31	1453.69
19	<i>Tasso1</i>	4.00%		2	58.79	58.15	116.94	1394.90
20	<i>Tasso2</i>	5.00%		3	71.77	55.80	127.57	1323.12
21				4	85.28	52.92	138.20	1237.85
22	<i>Caso</i>	1		5	99.32	49.51	148.83	1138.53
23		9.3851		6	0.00	56.93	56.93	1138.53
24	<i>Rata1</i>	106.31		7	0.00	56.93	56.93	1138.53
25	<i>Rata2</i>	280.69		8	0.00	56.93	56.93	1138.53
26		3.5460		9	223.76	56.93	280.69	914.77
27	<i>Tax Val</i>	8.00%		10	263.02	45.74	308.76	651.75
28				11	304.24	32.59	336.82	347.52
29	<i>g</i>	10.00%		12	347.52	17.38	364.89	0.00

	A	B	C	I	J	K	L	M
16	<i>Capitale</i>	1500		<i>Anni</i>	<i>Capit</i>	<i>Inter</i>	<i>Rata</i>	<i>Residuo</i>
17	<i>Durata</i>	12		0				1500.00
18	<i>Durata2</i>	4		1	46.31	60.00	106.31	1453.69
19	<i>Tasso1</i>	4.00%		2	58.79	58.15	116.94	1394.90
20	<i>Tasso2</i>	5.00%		3	71.77	55.80	127.57	1323.12
21				4	85.28	52.92	138.20	1237.85
22	<i>Caso</i>	2		5	99.32	49.51	148.83	1138.53
23		9.3851		6	-56.93	56.93	0.00	1195.46
24	<i>Rata1</i>	106.31		7	-59.77	59.77	0.00	1255.23
25	<i>Rata2</i>	324.93		8	-62.76	62.76	0.00	1317.99
26		3.5460		9	259.03	65.90	324.93	1058.96
27	<i>Tax Val</i>	8.00%		10	304.47	52.95	357.42	754.49
28				11	352.19	37.72	389.92	402.29
29	<i>g</i>	10.00%		12	402.29	20.11	422.41	0.00

	A	B	C	I	N	O	P	Q
16	<i>Capitale</i>	1500		<i>Anni</i>	<i>Nprop</i>	<i>Usuf</i>	<i>Valore</i>	<i>Grado</i>
17	<i>Durata</i>	12		0	782.78	392.68	1175.47	33.41%
18	<i>Durata2</i>	4		1	799.10	364.10	1163.20	31.30%
19	<i>Tasso1</i>	4.00%		2	804.23	335.08	1139.31	29.41%
20	<i>Tasso2</i>	5.00%		3	796.80	306.09	1102.89	27.75%
21				4	775.27	277.65	1052.92	26.37%
22	<i>Caso</i>	1		5	737.97	250.35	988.32	25.33%
23		9.3851		6	797.01	213.45	1010.46	21.12%
24	<i>Rata1</i>	106.31		7	860.77	173.60	1034.37	16.78%
25	<i>Rata2</i>	280.69		8	929.63	130.56	1060.19	12.32%
26		3.5460		9	780.24	84.08	864.32	9.73%
27	<i>Tax Val</i>	8.00%		10	579.64	45.07	624.71	7.21%
28				11	321.77	16.09	337.86	4.76%
29	<i>g</i>	10.00%		12				

	A	B	C	I	N	O	P	Q
16	Capitale	1500		Anni	Nprop	Usuf	Valore	Grado
17	Durata	12		0	757.29	408.62	1165.91	35.05%
18	Durata2	4		1	771.57	381.31	1152.87	33.07%
19	Tasso1	4.00%		2	774.50	353.66	1128.16	31.35%
20	Tasso2	5.00%		3	764.69	326.16	1090.85	29.90%
21				4	740.58	299.33	1039.91	28.78%
22	Caso	2		5	700.51	273.76	974.27	28.10%
23		9.3851		6	813.48	238.73	1052.22	22.69%
24	Rata1	106.31		7	938.33	198.06	1136.39	17.43%
25	Rata2	324.93		8	1076.16	151.14	1227.30	12.32%
26		3.5460		9	903.22	97.34	1000.56	9.73%
27	Tax Val	8.00%		10	671.01	52.18	723.18	7.21%
28				11	372.49	18.62	391.12	4.76%
29	g	10.00%		12				

	A	B
16	Capitale	1500
17	Durata	12
18	Durata2	4
19	Tasso1	0.04
20	Tasso2	=B19+0.01
21		
22	Caso	2
23		=(1-(1+B19)^-B17)/B19
24	Rata1	=M17/(((1-B29)*B23+B29*((1+B19)*B23-B17*(1+B19)^-B17)/B19)
25	Rata2	=M25/(((1-B29)*B26+B29*((1+B20)*B26-B18*(1+B20)^-B18)/B20)
26		=(1-(1+B20)^-B18)/B20
27	Tax Val	0.08
28		
29	g	0.1

	D	E	F	G	H
16	Anni	Capit	Inter	Rata	Residuo
17	0				=B16
18	1	=G18-F18	=H17*\$B\$19	=B\$24*(1+B\$29*(D18-1))	=H17-E18
19	2	=G19-F19	=H18*\$B\$19	=B\$24*(1+B\$29*(D19-1))	=H18-E19
20	3	=G20-F20	=H19*\$B\$19	=B\$24*(1+B\$29*(D20-1))	=H19-E20

	I	J	K	L	M
16	Anni	Capit	Inter	Rata	Residuo
17	0				=B16
18	1	=L18-K18	=M17*\$B\$19	=B\$24*(1+B\$29*(I18-1))	=M17-J18
19	2	=L19-K19	=M18*\$B\$19	=B\$24*(1+B\$29*(I19-1))	=M18-J19
20	3	=L20-K20	=M19*\$B\$19	=B\$24*(1+B\$29*(I20-1))	=M19-J20
21	4	=L21-K21	=M20*\$B\$19	=B\$24*(1+B\$29*(I21-1))	=M20-J21
22	5	=L22-K22	=M21*\$B\$19	=B\$24*(1+B\$29*(I22-1))	=M21-J22
23	6	=L23-K23	=M22*\$B\$20	=K23*(B\$22=1)	=M22-J23
24	7	=L24-K24	=M23*\$B\$20	=K24*(B\$22=1)	=M23-J24
25	8	=L25-K25	=M24*\$B\$20	=K25*(B\$22=1)	=M24-J25
26	9	=L26-K26	=M25*\$B\$20	=B\$25*(1+B\$29*(I26-9))	=M25-J26
27	10	=L27-K27	=M26*\$B\$20	=B\$25*(1+B\$29*(I27-9))	=M26-J27
28	11	=L28-K28	=M27*\$B\$20	=B\$25*(1+B\$29*(I28-9))	=M27-J28
29	12	=L29-K29	=M28*\$B\$20	=B\$25*(1+B\$29*(I29-9))	=M28-J29

	N	O	P	Q
16	Nprop	Usuf	Valore	Grado
17	=(N18+J18)/(1+\$B\$27)	=(O18+K18)/(1+\$B\$27)	=(P18+L18)/(1+\$B\$27)	=O17/P17
18	=(N19+J19)/(1+\$B\$27)	=(O19+K19)/(1+\$B\$27)	=(P19+L19)/(1+\$B\$27)	=O18/P18
19	=(N20+J20)/(1+\$B\$27)	=(O20+K20)/(1+\$B\$27)	=(P20+L20)/(1+\$B\$27)	=O19/P19

Esercizio A (proseguimento)

Ripetizione dell'esercizio nell'ipotesi che le rate costituiscano una progressione geometrica di ragione relativa g ($=10\%$)

$$D_0 = P$$

$$\prod_{t=1}^T R_t = \frac{D_0(1+g)^{t-1}}{(Ga)_{\overline{T}|i}}, \quad \prod_{t=1}^T I_t = D_{t-1}i, \quad \prod_{t=1}^T C_t = R_t - I_t, \quad \prod_{t=1}^T D_t = D_{t-1} - C_t$$

$$D_0 = P$$

$$\sum_{t=1}^R R_t = \frac{D_0 (1+g)^{t-1}}{(Ga)_{\overline{T}|i}^{(g)}}, \quad \sum_{t=R+1}^{R+S} R_t = D_{t-1} \hat{i} \quad (sw = 1)$$

$$\sum_{t=R+S+1}^T R_t = \frac{D_{R+S} (1+g)^{t-R-S-1}}{(Ga)_{\overline{T-R-S}|i}^{(g)}}$$

$$\sum_{t=1}^R I_t = D_{t-1} i, \quad \sum_{t=R+1}^T I_t = D_{t-1} \hat{i}$$

$$\sum_{t=1}^T C_t = R_t - I_t, \quad \sum_{t=1}^T D_t = D_{t-1} - C_t$$

	A	B	C	D	E	F	G	H
31	Capitale	1500		Anni	Capit	Inter	Rata	Residuo
32	Durata	12		0				1500.00
33	Durata2	4		1	33.73	60.00	93.73	1466.27
34	Tasso1	4.00%		2	44.45	58.65	103.10	1421.83
35	Tasso2	5.00%		3	56.53	56.87	113.41	1365.29
36				4	70.14	54.61	124.75	1295.16
37	Caso	1		5	85.42	51.81	137.22	1209.74
38		-5.4545%		6	102.56	48.39	150.95	1107.18
39	Rata1	93.73		7	121.75	44.29	166.04	985.43
40	Rata2	295.75		8	143.23	39.42	182.64	842.20
41		-4.5455%		9	167.22	33.69	200.91	674.98
42	Tax Val	8.00%		10	194.00	27.00	221.00	480.98
43				11	223.86	19.24	243.10	257.12
44	g	10.00%		12	257.12	10.28	267.41	0.00

	A	B	C	I	J	K	L	M
31	Capitale	1500		Anni	Capit	Inter	Rata	Residuo
32	Durata	12		0				1500.00
33	Durata2	4		1	33.73	60.00	93.73	1466.27
34	Tasso1	4.00%		2	44.45	58.65	103.10	1421.83
35	Tasso2	5.00%		3	56.53	56.87	113.41	1365.29
36				4	70.14	54.61	124.75	1295.16
37	Caso	1		5	85.42	51.81	137.22	1209.74
38		-5.4545%		6	0.00	60.49	60.49	1209.74
39	Rata1	93.73		7	0.00	60.49	60.49	1209.74
40	Rata2	295.75		8	0.00	60.49	60.49	1209.74
41		-4.5455%		9	235.27	60.49	295.75	974.47
42	Tax Val	8.00%		10	276.60	48.72	325.33	697.87
43				11	322.97	34.89	357.86	374.90
44	g	10.00%		12	374.90	18.75	393.65	0.00

	A	B	C	I	J	K	L	M
31	Capitale	1500		Anni	Capit	Inter	Rata	Residuo
32	Durata	12		0				1500.00
33	Durata2	4		1	33.73	60.00	93.73	1466.27
34	Tasso1	4.00%		2	44.45	58.65	103.10	1421.83
35	Tasso2	5.00%		3	56.53	56.87	113.41	1365.29
36				4	70.14	54.61	124.75	1295.16
37	Caso	2		5	85.42	51.81	137.22	1209.74
38		-5.4545%		6	-60.49	60.49	0.00	1270.23
39	Rata1	93.73		7	-63.51	63.51	0.00	1333.74
40	Rata2	342.37		8	-66.69	66.69	0.00	1400.43
41		-4.5455%		9	272.35	70.02	342.37	1128.08
42	Tax Val	8.00%		10	320.20	56.40	376.61	807.87
43				11	373.88	40.39	414.27	434.00
44	g	10.00%		12	434.00	21.70	455.70	0.00

	A	B	C	D	N	O	P	Q
31	Capitale	1500		Anni	Nprop	Usuf	Valore	Grado
32	Durata	12		0	757.10	407.71	1164.82	35.00%
33	Durata2	4		1	783.95	380.33	1164.28	32.67%
34	Tasso1	4.00%		2	802.22	352.11	1154.32	30.50%
35	Tasso2	5.00%		3	809.86	323.40	1133.26	28.54%
36				4	804.51	294.66	1099.17	26.81%
37	Caso	1		5	783.46	266.43	1049.88	25.38%
38		-5.4545%		6	846.13	227.25	1073.39	21.17%
39	Rata1	93.73		7	913.82	184.95	1098.77	16.83%
40	Rata2	295.75		8	986.93	139.26	1126.19	12.37%
41		-4.5455%		9	830.62	89.91	920.53	9.77%
42	Tax Val	8.00%		10	620.46	48.38	668.84	7.23%
43				11	347.13	17.36	364.49	4.76%
44	g	10.00%		12				

	A	B	C	D	N	O	P	Q
31	Capitale	1500		Anni	Nprop	Usuf	Valore	Grado
32	Durata	12		0	729.95	424.69	1154.63	36.78%
33	Durata2	4		1	754.62	398.66	1153.28	34.57%
34	Tasso1	4.00%		2	770.54	371.90	1142.44	32.55%
35	Tasso2	5.00%		3	775.65	344.78	1120.43	30.77%
36				4	767.56	317.75	1085.32	29.28%
37	Caso	2		5	743.55	291.37	1034.92	28.15%
38		-5.4545%		6	863.52	254.19	1117.71	22.74%
39	Rata1	93.73		7	996.12	211.01	1207.13	17.48%
40	Rata2	342.37		8	1142.49	161.21	1303.70	12.37%
41		-4.5455%		9	961.54	104.08	1065.63	9.77%
42	Tax Val	8.00%		10	718.26	56.01	774.27	7.23%
43				11	401.85	20.09	421.94	4.76%
44	g	10.00%		12				

	A	B
31	Capitale	1500
32	Durata	12
33	Durata2	4
34	Tasso1	0.04
35	Tasso2	=B34+0.01
36		
37	Caso	2
38		=(B34-B44)/(1+B44)
39	Rata1	=M32*(B34-B44)/(1-(1+B38)^-B32)
40	Rata2	=M40*(B35-B44)/(1-(1+B41)^-B33)
41		=(B35-B44)/(1+B44)
42	Tax Val	0.08
43		
44	g	0.1

	D	E	F	G	H
31	Anni	Capit	Inter	Rata	Residuo
32	0				=B31
33	1	=G33-F33	=H32*\$B\$19	=B\$39*(1+B\$44)^(D33-1)	=H32-E33
34	2	=G34-F34	=H33*\$B\$19	=B\$39*(1+B\$44)^(D34-1)	=H33-E34
35	3	=G35-F35	=H34*\$B\$19	=B\$39*(1+B\$44)^(D35-1)	=H34-E35

	I	J	K	L	M
31	Anni	Capit	Inter	Rata	Residuo
32	0				=B31
33	1	=L33-K33	=M32*\$B\$19	=B\$39*(1+B\$44)^(I33-1)	=M32-J33
34	2	=L34-K34	=M33*\$B\$19	=B\$39*(1+B\$44)^(I34-1)	=M33-J34
35	3	=L35-K35	=M34*\$B\$19	=B\$39*(1+B\$44)^(I35-1)	=M34-J35
36	4	=L36-K36	=M35*\$B\$19	=B\$39*(1+B\$44)^(I36-1)	=M35-J36
37	5	=L37-K37	=M36*\$B\$19	=B\$39*(1+B\$44)^(I37-1)	=M36-J37
38	6	=L38-K38	=M37*\$B\$20	=K38*(B\$37=1)	=M37-J38
39	7	=L39-K39	=M38*\$B\$20	=K39*(B\$37=1)	=M38-J39
40	8	=L40-K40	=M39*\$B\$20	=K40*(B\$37=1)	=M39-J40
41	9	=L41-K41	=M40*\$B\$20	=B\$40*(1+B\$44)^(I41-9)	=M40-J41
42	10	=L42-K42	=M41*\$B\$20	=B\$40*(1+B\$44)^(I42-9)	=M41-J42
43	11	=L43-K43	=M42*\$B\$20	=B\$40*(1+B\$44)^(I43-9)	=M42-J43
44	12	=L44-K44	=M43*\$B\$20	=B\$40*(1+B\$44)^(I44-9)	=M43-J44

	N	O	P	Q
31	<i>Nprop</i>	<i>Usuf</i>	<i>Valore</i>	<i>Grado</i>
32	$= (N_{33} + J_{33}) / (1 + S_{33})$	$= (O_{33} + K_{33}) / (1 + S_{33})$	$= (P_{33} + L_{33}) / (1 + S_{33})$	$= O_{32} / P_{32}$
33	$= (N_{34} + J_{34}) / (1 + S_{34})$	$= (O_{34} + K_{34}) / (1 + S_{34})$	$= (P_{34} + L_{34}) / (1 + S_{34})$	$= O_{33} / P_{33}$
34	$= (N_{35} + J_{35}) / (1 + S_{35})$	$= (O_{35} + K_{35}) / (1 + S_{35})$	$= (P_{35} + L_{35}) / (1 + S_{35})$	$= O_{34} / P_{34}$

Esercizio B

Al tempo iniziale 0 un individuo contrae un mutuo di ammontare D_0 ($=10000$), con obbligo di rimborso, secondo la metodologia dell'ammortamento italiano, in S ($=7$) anni ed al tasso annuo d'interesse i_1 ($=6\%$). Immediatamente dopo il versamento della P -sima ($=4$) rata, l'individuo accetta la modifica delle modalità di rimborso del mutuo, con raddoppio del periodo residuo di pagamento, cambio del tasso annuo di interesse da i_1 a i_2 ($=8\%$) e suddivisione del debito residuo in due parti:

- una parte del debito residuo pari al $H\%$ ($=60\%$), da rimborsare secondo la metodologia dell'ammortamento francese,
- una parte del citato debito residuo pari al $(100-H)\%$, da rimborsare secondo la metodologia dell'ammortamento a mutuo puro.
- Si chiede di determinare:
- il piano di ammortamento italiano originario relativo al periodo di S anni,
- il piano di ammortamento francese relativo al periodo di $2(S-P)$ anni,
- il piano di ammortamento a mutuo puro relativo al periodo di $2(S-P)$ anni,

- *Il piano di ammortamento completo relativo al periodo di $2S-P$ anni e composto dai primi P anni del piano originario e, per i successivi $2(S-P)$ anni, dalla somma dei due piani (francese e a mutuo puro),*
- *la nuda proprietà, l'usufrutto, il valore del prestito e il conseguente indice del grado di capitalizzazione, relativamente al piano di ammortamento completo ed al tasso annuo di interesse i_3 (=10%).*

Nota:

Considerare fisse le grandezze (di tipo durata) S e P e variabili le altre grandezze (di tipo finanziario) D_0, i_1, i_2, i_3 ed H .

- *Dati del problema*

	A	B
1	Deb	10000
2	Dur	7
3	Mod	4
4	i_1	6%
5	i_2	8%
6	i_3	10%
7	H	60%

- *Ammortamento italiano relativo al periodo di S anni*

$$\prod_{t=1}^S C_t = \frac{D_0}{S}, \quad \prod_{t=1}^S D_t = D_{t-1} - C_t, \quad \prod_{t=1}^S I_t = D_{t-1} i_1, \quad \prod_{t=1}^S R_t = C_t + I_t$$

	A	B	C	D	E
9	Anni	Capit	Inter	Rate	Residuo
10	0				10000.00
11	1	1428.57	600.00	2028.57	8571.43
12	2	1428.57	514.29	1942.86	7142.86
13	3	1428.57	428.57	1857.14	5714.29
14	4	1428.57	342.86	1771.43	4285.71
15	5	1428.57	257.14	1685.71	2857.14
16	6	1428.57	171.43	1600.00	1428.57
17	7	1428.57	85.71	1514.29	0.00

	A	B	C	D	E
9	Anni	Capit	Inter	Rate	Residuo
10	0				=B1
11	1	=E10/B2	=E10*\$B\$4	=B11+C11	=E10-B11
12	2	=B11	=E11*\$B\$4	=B12+C12	=E11-B12
13	3	=B12	=E12*\$B\$4	=B13+C13	=E12-B13

- **Ammortamento francese relativo al periodo di 2(S-P) anni**

$$D_p^F = HD_p, \quad \prod_{t=P+1}^{2S-P} R_t^F = \frac{D_p^F}{a_{\overline{2(S-P)}|i_2}}, \quad \prod_{t=P+1}^{2S-P} D_t^F = D_{t-1}^F - C_t^F$$

$$\prod_{t=P+1}^{2S-P} I_t^F = D_{t-1}^F i_2, \quad \prod_{t=P+1}^{2S-P} C_t^F = R_t^F - I_t^F$$

	A	B	C	D	E
19	Anni	Capit	Inter	Rate	Residuo
20	4				2571.43
21	5	350.53	205.71	556.24	2220.90
22	6	378.57	177.67	556.24	1842.34
23	7	408.85	147.39	556.24	1433.48
24	8	441.56	114.68	556.24	991.92
25	9	476.89	79.35	556.24	515.04
26	10	515.04	41.20	556.24	0.00

	A	B	C	D	E
19	Anni	Capit	Inter	Rate	Residuo
20	4				=E14*B7
21	5	=D21-C21	=E20*\$B\$5	=E20*B5/(1-(1+B5)^-(2*(B2-B3)))	=E20-B21
22	6	=D22-C22	=E21*\$B\$5	=D21	=E21-B22
23	7	=D23-C23	=E22*\$B\$5	=D22	=E22-B23

- **Ammortamento a mutuo puro relativo al periodo di 2 (S-P) anni**

$$D_P^U = (1-H)D_P, \quad \begin{cases} \sum_{t=P+1}^{2S-P-1} C_t^U = 0 \\ C_{2S-P}^U = D_P^U \end{cases}, \quad \sum_{t=P+1}^{2S-P} D_t^U = D_{t-1}^U - C_t^U$$

$$\sum_{t=P+1}^{2S-P} I_t^U = D_{t-1}^U i_2, \quad \sum_{t=P+1}^{2S-P} R_t^U = C_t^U + I_t^U$$

	A	B	C	D	E
28	Anni	Capit	Inter	Rate	Residuo
29	4				1714.29
30	5	0.00	137.14	137.14	1714.29
31	6	0.00	137.14	137.14	1714.29
32	7	0.00	137.14	137.14	1714.29
33	8	0.00	137.14	137.14	1714.29
34	9	0.00	137.14	137.14	1714.29
35	10	1714.29	137.14	1851.43	0.00

	A	B	C	D	E
28	Anni	Capit	Inter	Rate	Residuo
29	4				=E14*(1-B7)
30	5	0	=E29*\$B\$5	=B30+C30	=E29-B30
34	9	0	=E33*\$B\$5	=B34+C34	=E33-B34
35	10	=E29	=E34*\$B\$5	=B35+C35	=E34-B35

- **Ammortamento completo relativo al periodo di 2S-P anni**

$$\left\{ \begin{array}{l} \prod_{t=1}^P C_t^C = C_t, I_t^C = I_t, R_t^C = R_t, D_t^C = D_t \\ \prod_{t=P+1}^{2S-P} C_t^C = C_t^F + C_t^U, I_t^C = I_t^F + I_t^U, R_t^C = R_t^F + R_t^U, D_t^C = D_t^F + D_t^U \end{array} \right.$$

	G	H	I	J	K
9	Anni	Capit	Inter	Rate	Residuo
10	0				10000.00
11	1	1428.57	600.00	2028.57	8571.43
12	2	1428.57	514.29	1942.86	7142.86
13	3	1428.57	428.57	1857.14	5714.29
14	4	1428.57	342.86	1771.43	4285.71
15	5	350.53	342.86	693.38	3935.19
16	6	378.57	314.82	693.38	3556.62
17	7	408.85	284.53	693.38	3147.77
18	8	441.56	251.82	693.38	2706.21
19	9	476.89	216.50	693.38	2229.32
20	10	2229.32	178.35	2407.67	0.00

	G	H	I	J	K	L
9	Anni	Capit	Inter	Rate	Residuo	Coeff
10	0				=E10	
11	1	=B11	=C11	=D11	=E11	=(1+\$B\$6)^-G11
14	4	=B14	=C14	=D14	=E14	=(1+\$B\$6)^-G14
15	5	=B21+B30	=C21+C30	=D21+D30	=E21+E30	=(1+\$B\$6)^-G15
20	10	=B26+B35	=C26+C35	=D26+D35	=E26+E35	=(1+\$B\$6)^-G20

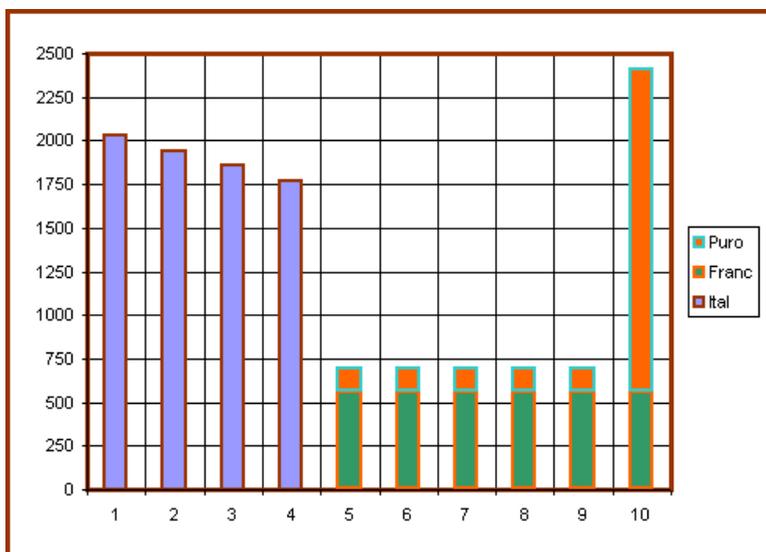
- *Nuda proprietà, usufrutto, valore del prestito e conseguente indice del grado di capitalizzazione*

$$NPr = \sum_{t=1}^{2S-P} C_t^C (1+i_3)^{-t} , \quad Usu = \sum_{t=1}^{2S-P} I_t^C (1+i_3)^{-t}$$

$$Val = \sum_{t=1}^{2S-P} R_t^C (1+i_3)^{-t} , \quad GrC = \frac{Usu}{Val} = \frac{\sum_{t=1}^{2S-P} I_t^C (1+i_3)^{-t}}{\sum_{t=1}^{2S-P} R_t^C (1+i_3)^{-t}}$$

	G	L	M	N	O	P
9	Anni	Coeff	Capit	Inter	Rate	
10	0					
11	1	0.9091	1298.70	545.45	1844.16	
12	2	0.8264	1180.64	425.03	1605.67	
13	3	0.7513	1073.31	321.99	1395.30	
14	4	0.6830	975.73	234.18	1209.91	
15	5	0.6209	217.65	212.89	430.54	
16	6	0.5645	213.69	177.70	391.40	
17	7	0.5132	209.81	146.01	355.81	
18	8	0.4665	205.99	117.48	323.47	
19	9	0.4241	202.25	91.82	294.06	
20	10	0.3855	859.50	68.76	928.26	
21			6437.26	2341.31	8778.57	26.7%
22			NPro	Usu	Val	Grado

	G	L	M	N	O	P
9	Anni	Coeff	Capit	Inter	Rate	
10	0					
11	1	=(1+\$B\$6)^-G11	=H11*\$L11	=I11*\$L11	=J11*\$L11	
12	2	=(1+\$B\$6)^-G12	=H12*\$L12	=I12*\$L12	=J12*\$L12	
19	9	=(1+\$B\$6)^-G19	=H19*\$L19	=I19*\$L19	=J19*\$L19	
20	10	=(1+\$B\$6)^-G20	=H20*\$L20	=I20*\$L20	=J20*\$L20	
21			=SOMMA(M11:M20)	=SOMMA(N11:N20)	=SOMMA(O11:O20)	=N21/O21
22			NPro	Usu	Val	Grado



Esercizio C

Al tempo iniziale 0 un individuo contrae un mutuo di ammontare D_0 ($=10000$), con obbligo di rimborso, secondo la metodologia dell'ammortamento di tipo francese, in S ($=10$) anni ed al tasso annuo effettivo d'interesse i ($=4\%$). Immediatamente dopo il versamento della P -sima ($=6$) rata, l'individuo propone, a partire dalla successiva rata, di pagare, quale quota capitale, una porzione, pari ad $H\%$ (60%), della quota capitale dell'ammortamento originario, con accettazione del raddoppio del tasso d'interesse. Il debito residuo non rimborsato alla scadenza S , dovrà essere estinto con un ammortamento di tipo italiano, per una durata pari ad ulteriori $S-P$ anni e ad un tasso annuo effettivo annuo d'interesse pari alla media aritmetica dei due tassi precedentemente utilizzati.

Si chiede di determinare:

- *il piano di ammortamento francese originario relativo al periodo di S anni,*
- *il piano di ammortamento modificato (a quote capitale prefissate) relativo al periodo di $S-P$ anni,*
- *il piano di ammortamento italiano residuo relativo al periodo $S-P$ anni,*
- *Il piano di ammortamento completo relativo al periodo di $2S-P$ anni,*
- *con riguardo al piano di ammortamento completo, la nuda proprietà, l'usufrutto, il valore del prestito e l'indice del grado di capitalizzazione, ai diversi tempi di evoluzione del processo di ammortamento ed al tasso annuo medio utilizzato nel terzo periodo.*

Nota:

Considerare fisse le grandezze (di tipo durata) S e P e variabili le altre grandezze (di tipo finanziario) D_0 , i ed H .

- *Dati del problema*

	A	B	C	D
1	Deb	10000		
2	Dur	10		
3	Mod	6		
4	H	60%		
5	I	4.00%	8.00%	6.00%

	A	C	D
5	I	=B5*2	=(B5+C5)/2

- *Ammortamento francese relativo al periodo di S anni*

$$\prod_{t=1}^S R_t = \frac{D_0}{a_{\overline{S}|i}} \quad , \quad \prod_{t=1}^S D_t = D_{t-1} - C_t$$

$$\prod_{t=1}^S I_t = D_{t-1} i \quad , \quad \prod_{t=1}^S C_t = R_t - I_t$$

	A	B	C	D	E
8	Anni	Capit	Inter	Rate	Residuo
9	0				10000.00
10	1	832.91	400.00	1232.91	9167.09
11	2	866.23	366.68	1232.91	8300.86
12	3	900.87	332.03	1232.91	7399.99
13	4	936.91	296.00	1232.91	6463.08
14	5	974.39	258.52	1232.91	5488.69
15	6	1013.36	219.55	1232.91	4475.33
16	7	1053.90	179.01	1232.91	3421.44
17	8	1096.05	136.86	1232.91	2325.38
18	9	1139.89	93.02	1232.91	1185.49
19	10	1185.49	47.42	1232.91	0.00

	A	B	C	D	E
8	Anni	Capit	Inter	Rate	Residuo
9	0				=B1
10	1	=D10-C10	=E9*\$B\$5	=B1*B5/(1-(1+B5)^-B2)	=E9-B10
11	2	=D11-C11	=E10*\$B\$5	=D10	=E10-B11
12	3	=D12-C12	=E11*\$B\$5	=D11	=E11-B12

- *Ammortamento modificato relativo al periodo di S-P anni*

$$D_P^M = D_P \quad , \quad \prod_{t=P+1}^S C_t^M = HK_t \quad , \quad \prod_{t=P+1}^S D_t^M = D_{t-1}^M - C_t^M$$

$$\prod_{t=P+1}^S I_t^M = D_{t-1}^M 2i \quad , \quad \prod_{t=P+1}^S R_t^M = C_t^M + I_t^M$$

	A	B	C	D	E
21	Anni	Capit	Inter	Rate	Residuo
22	6				4475.33
23	7	632.34	358.03	990.36	3842.99
24	8	657.63	307.44	965.07	3185.36
25	9	683.94	254.83	938.77	2501.43
26	10	711.29	200.11	911.41	1790.13

	A	B	C	D	E
21	Anni	Capit	Inter	Rate	Residuo
22	6				=E15
23	7	=B16*\$B\$4	=E22*\$C\$5	=B23+C23	=E22-B23
24	8	=B17*\$B\$4	=E23*\$C\$5	=B24+C24	=E23-B24
25	9	=B18*\$B\$4	=E24*\$C\$5	=B25+C25	=E24-B25
26	10	=B19*\$B\$4	=E25*\$C\$5	=B26+C26	=E25-B26

- *Ammortamento italiano relativo al periodo di S-P anni*

$$D'_s = D_s^M, \quad \prod_{t=S+1}^{S+P} K'_t = \frac{D'_s}{S-P}, \quad \prod_{t=S+1}^{S+P} D'_t = D'_{t-1} - K'_t$$

$$\prod_{t=S+1}^{S+P} I'_t = D'_{t-1} \frac{3}{2} i, \quad \prod_{t=S+1}^{S+P} R'_t = K'_t + I'_t$$

	A	B	C	D	E
28	Anni	Capit	Inter	Rate	Residuo
29	10				1790.13
30	11	447.53	107.41	554.94	1342.60
31	12	447.53	80.56	528.09	895.07
32	13	447.53	53.70	501.24	447.53
33	14	447.53	26.85	474.39	0.00

	A	B	C	D	E
28	Anni	Capit	Inter	Rate	Residuo
29	10				=E26
30	11	=E29/(B2-B3)	=E29*\$D\$5	=B30+C30	=E29-B30
31	12	=B30	=E30*\$D\$5	=B31+C31	=E30-B31
32	13	=B31	=E31*\$D\$5	=B32+C32	=E31-B32
33	14	=B32	=E32*\$D\$5	=B33+C33	=E32-B33

- *Ammortamento completo relativo al periodo di 2S-P anni*

$$\left\{ \begin{array}{l} \sum_{t=1}^P C_t^C = C_t \\ \sum_{t=P+1}^S C_t^C = C_t^M \\ \sum_{t=S+1}^{S+P} C_t^C = C_t^I \end{array} \right\}, \left\{ \begin{array}{l} \sum_{t=1}^P I_t^C = I_t \\ \sum_{t=P+1}^S I_t^C = I_t^M \\ \sum_{t=S+1}^{S+P} I_t^C = I_t^I \end{array} \right\}, \left\{ \begin{array}{l} \sum_{t=1}^P R_t^C = R_t \\ \sum_{t=P+1}^S R_t^C = R_t^M \\ \sum_{t=S+1}^{S+P} R_t^C = R_t^I \end{array} \right\}, \left\{ \begin{array}{l} \sum_{t=1}^P D_t^C = D_t \\ \sum_{t=P+1}^S D_t^C = D_t^M \\ \sum_{t=S+1}^{S+P} D_t^C = D_t^I \end{array} \right\}$$

	G	H	I	J	K
8	Anni	Capit	Inter	Rate	Residuo
9	0				10000.00
10	1	832.91	400.00	1232.91	9167.09
11	2	866.23	366.68	1232.91	8300.86
12	3	900.87	332.03	1232.91	7399.99
13	4	936.91	296.00	1232.91	6463.08
14	5	974.39	258.52	1232.91	5488.69
15	6	1013.36	219.55	1232.91	4475.33
16	7	632.34	358.03	990.36	3842.99
17	8	657.63	307.44	965.07	3185.36
18	9	683.94	254.83	938.77	2501.43
19	10	711.29	200.11	911.41	1790.13
20	11	447.53	107.41	554.94	1342.60
21	12	447.53	80.56	528.09	895.07
22	13	447.53	53.70	501.24	447.53
23	14	447.53	26.85	474.39	0.00

	G	H	I	J	K
8	Anni	Capit	Inter	Rate	Residuo
9	0				=E9
10	1	=B10	=C10	=D10	=E10
11	2	=B11	=C11	=D11	=E11
12	3	=B12	=C12	=D12	=E12

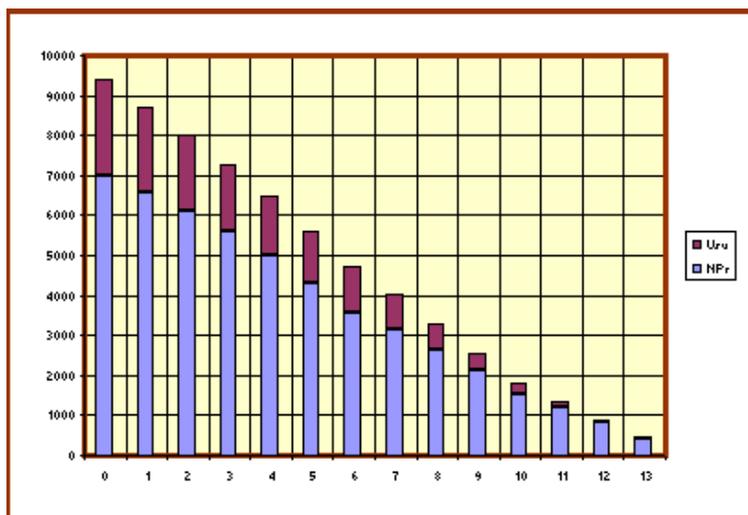
- *Nuda proprietà, usufrutto, valore del prestito ed indice del grado di capitalizzazione*

$$\left\{ \begin{array}{l}
 NPr_{S+P-1} = \frac{C_{S+P}^C}{1 + \frac{3}{2}i} \\
 \prod_{t=S+P-2}^0 \underset{(-1)}{NPr_t} = \frac{NPr_{t+1} + C_{t+1}^C}{1 + \frac{3}{2}i}
 \end{array} \right\}, \quad \left\{ \begin{array}{l}
 Usu_{S+P-1} = \frac{I_{S+P}^C}{1 + \frac{3}{2}i} \\
 \prod_{t=S+P-2}^0 \underset{(-1)}{Usu_t} = \frac{Usu_{t+1} + I_{t+1}^C}{1 + \frac{3}{2}i}
 \end{array} \right\}$$

$$\left\{ \begin{array}{l}
 Val_{S+P-1} = \frac{R_{S+P}^C}{1 + \frac{3}{2}i} \\
 \prod_{t=S+P-2}^0 \underset{(-1)}{Val_t} = \frac{Val_{t+1} + R_{t+1}^C}{1 + \frac{3}{2}i}
 \end{array} \right\}, \quad \prod_{t=0}^{S+P-1} GrC_t = \frac{Usu_t}{Val_t}$$

	G	L	M	N	O
8	Anni	Capit	Inter	Rate	
9	0	6998.79	2392.15	9390.94	25.5%
10	1	6585.81	2135.68	8721.49	24.5%
11	2	6114.73	1897.14	8011.87	23.7%
12	3	5580.74	1678.93	7259.67	23.1%
13	4	4978.68	1483.66	6462.34	23.0%
14	5	4303.01	1314.16	5617.17	23.4%
15	6	3547.83	1173.46	4721.29	24.9%
16	7	3128.36	885.84	4014.21	22.1%
17	8	2658.43	631.56	3289.99	19.2%
18	9	2134.00	414.62	2548.62	16.3%
19	10	1550.75	239.38	1790.13	13.4%
20	11	1196.26	146.34	1342.60	10.9%
21	12	820.50	74.56	895.07	8.3%
22	13	422.20	25.33	447.53	5.7%
23	14	NPro	Usu	Val	Grado

	L	M	N	O
8	<i>Capit</i>	<i>Inter</i>	<i>Rate</i>	
9	$= (L_{10} + H_{10}) / (1 + \$D\$5)$	$= (M_{10} + I_{10}) / (1 + \$D\$5)$	$= (N_{10} + J_{10}) / (1 + \$D\$5)$	$= M9/N9$
10	$= (L_{11} + H_{11}) / (1 + \$D\$5)$	$= (M_{11} + I_{11}) / (1 + \$D\$5)$	$= (N_{11} + J_{11}) / (1 + \$D\$5)$	$= M_{10}/N_{10}$
11	$= (L_{12} + H_{12}) / (1 + \$D\$5)$	$= (M_{12} + I_{12}) / (1 + \$D\$5)$	$= (N_{12} + J_{12}) / (1 + \$D\$5)$	$= M_{11}/N_{11}$



Esercizio D

Un individuo contrae al tempo iniziale 0 un mutuo di ammontare pari a D_0 (=10000), da rimborsare secondo la metodologia dell'ammortamento francese, per una durata pari ad N (=8) anni e un tasso annuo effettivo d'interesse pari a i (=13.75%).

Le rate di tale ammortamento vengono indicizzate, al momento del pagamento, considerando:

- i cambi **Ecu/Lira** (rilevati all'inizio di ogni periodo annuo, a partire dal tempo iniziale), forniti dal vettore $E_{t=0,1,\dots,N}$ (con $N+1$ componenti),*
- i tassi annui di inflazione, dati dal vettore $U_{t=1,2,\dots,N}$ (con N componenti),*

secondo le seguenti modalità:

- determinazione per ciascun anno della rata **modificata** legata all'Ecu, ottenuta come prodotto tra la rata base annuale ed il rapporto tra il cambio **Ecu/Lira** dello stesso anno e quello vigente all'inizio dell'operazione,*
- pagamento per ciascun anno della rata **modificata**, sino ad una rata **massima** legata all'inflazione, ottenuta come prodotto tra la rata base ed il coefficiente totale d'inflazione dell'anno corrente rispetto alla situazione vigente all'inizio dell'operazione,*
- contabilizzazione, al tempo finale dell'operazione, delle eventuali differenze tra le rate modificate e gli importi pagati, in base ad un tasso annuo pari al tasso interno dell'operazione, calcolato con riguardo alle rate **modificate**,*

- *pagamento del montante residuale al tempo N , tramite un nuovo ammortamento di tipo italiano, di durata pari ad N ed al tasso interno calcolato.*

Si chiede di determinare:

- *il piano di ammortamento originario ed il vettore delle rate modificate,*
- *il vettore delle rate effettivamente pagate e l'ammontare, al tempo N , del nuovo debito da ammortizzare,*
- *il nuovo piano di ammortamento, verificando che il tasso interno dell'intera operazione coincide con il tasso interno precedentemente calcolato,*
- *il piano di ammortamento globale "a posteriori" calcolato in base al tasso interno unico.*

Nota:

Considerare fissa la grandezza (di tipo durata) N e variabili le altre grandezze (di tipo finanziario) D_0 , i , i cambi Ecu/Lira e i tassi di inflazione:

$$\sum_{t=0}^N E_t = (1254.05 \ 1293.12 \ 1754.01 \ 1701.32 \ 1483.76 \ 1582.50 \\ 1785.87 \ 1812.96 \ 1855.00)$$

$$\sum_{t=1}^N U_t = (5.25\% \ 5.01\% \ 4.72\% \ 4.37\% \ 4.55\% \ 5.12\% \ 5.57\% \ 6.31\%)$$

- *Dati del problema*

	A	B	C
1	Importo	D	10000
2	Inizio		0
3	Durata	N	8
4	Tasso	i	13.75%
5			1254.05
6			1293.12
7			1754.01
8			1701.32
9	Ecu/Lira	E[t]	1483.76
10			1582.50
11			1785.87
12			1812.96
13			1855.00
14			5.25%
15			5.01%
16			4.72%
17	Infla	U[t]	4.37%
18			4.55%
19			5.12%
20			5.57%
21			6.31%

- **Ammortamento francese relativo al periodo di N anni, vettore delle rate modificate e tasso interno di rendimento conseguente**

$$\begin{aligned} \prod_{t=1}^N R_t &= \frac{D_0}{a_{N|i}} \quad , \quad \prod_{t=1}^N D_t = D_{t-1} - C_t \\ \prod_{t=1}^N I_t &= D_{t-1} i \quad , \quad \prod_{t=1}^N K_t = R_t - I_t \quad , \quad \prod_{t=1}^N R_t^{mod} = R_t \frac{E_t}{E_0} \\ -D_0 + \sum_{t=1}^N R_t^{mod} (1 + \bar{i})^{-t} &= 0 \end{aligned}$$

	B	C	D	E	F	G
23	Anni	Capit	Inter	Rate	Resid	R mod.
24	0				10000.00	-10000.00
25	1	762.65	1375.00	2137.65	9237.35	2204.25
26	2	867.51	1270.14	2137.65	8369.84	2989.88
27	3	986.80	1150.85	2137.65	7383.04	2900.07
28	4	1122.48	1015.17	2137.65	6260.55	2529.21
29	5	1276.82	860.83	2137.65	4983.73	2697.53
30	6	1452.39	685.26	2137.65	3531.34	3044.19
31	7	1652.09	485.56	2137.65	1879.25	3090.37
32	8	1879.25	258.40	2137.65	0.00	3162.03
33	TIR					21.627%

	B	C	D	E	F	G
	Anni	Capit	Inter	Rate	Resid	R mod.
24	=C\$2				=C1	=F24
25	=B24+1	=E25-D25	=F24*C\$4	=F\$24*C\$4/(1-(1+C\$4)^(-C\$3))	=F24-C25	=E25*C6/C\$5
26	=B25+1	=E26-D26	=F25*C\$4	=F\$24*C\$4/(1-(1+C\$4)^(-C\$3))	=F25-C26	=E26*C7/C\$5
27	=B26+1	=E27-D27	=F26*C\$4	=F\$24*C\$4/(1-(1+C\$4)^(-C\$3))	=F26-C27	=E27*C8/C\$5
28	=B27+1	=E28-D28	=F27*C\$4	=F\$24*C\$4/(1-(1+C\$4)^(-C\$3))	=F27-C28	=E28*C9/C\$5
29	=B28+1	=E29-D29	=F28*C\$4	=F\$24*C\$4/(1-(1+C\$4)^(-C\$3))	=F28-C29	=E29*C10/C\$5
30	=B29+1	=E30-D30	=F29*C\$4	=F\$24*C\$4/(1-(1+C\$4)^(-C\$3))	=F29-C30	=E30*C11/C\$5
31	=B30+1	=E31-D31	=F30*C\$4	=F\$24*C\$4/(1-(1+C\$4)^(-C\$3))	=F30-C31	=E31*C12/C\$5
32	=B31+1	=E32-D32	=F31*C\$4	=F\$24*C\$4/(1-(1+C\$4)^(-C\$3))	=F31-C32	=E32*C13/C\$5
33	TIR					=TIR.COST(G24:G32)

- **Vettore della rate effettivamente pagate e ammontare del debito da ammortizzare**

$$\prod_{t=1}^N R_t^{pag} = \min \left(R_t^{mod}, R_t \prod_{h=1}^t (1 + U_h) \right)$$

$$M = \sum_{t=1}^N (R_t^{mod} - R_t^{pag}) (1 + \bar{i})^{N-t}$$

	B	H	I	J	K	L
23	Anni	Infla	R pag	Diff	Coeff	Mont
24	0	1.000000				
25	1	1.052500	2204.25	0.00	3.937400	0.00
26	2	1.105230	2362.60	627.28	3.237275	2030.69
27	3	1.157397	2474.11	425.96	2.661642	1133.74
28	4	1.207975	2529.21	0.00	2.188365	0.00
29	5	1.262938	2697.53	0.00	1.799243	0.00
30	6	1.327601	2837.95	206.24	1.479312	305.10
31	7	1.401548	2996.02	94.35	1.216270	114.75
32	8	1.489986	3162.03	0.00	1.000000	0.00
33	TIR					3584.28

	H	I	J	K	L
23	Infla	R pag	Diff	Coeff	Mont
24	1				
25	=H24*(1+C14)	=MIN(G25,E25*H25)	=G25-I25	=(1+G\$33)^(C\$3-B25)	=J25*K25
26	=H25*(1+C15)	=MIN(G26,E26*H26)	=G26-I26	=(1+G\$33)^(C\$3-B26)	=J26*K26
27	=H26*(1+C16)	=MIN(G27,E27*H27)	=G27-I27	=(1+G\$33)^(C\$3-B27)	=J27*K27
28	=H27*(1+C17)	=MIN(G28,E28*H28)	=G28-I28	=(1+G\$33)^(C\$3-B28)	=J28*K28
29	=H28*(1+C18)	=MIN(G29,E29*H29)	=G29-I29	=(1+G\$33)^(C\$3-B29)	=J29*K29
30	=H29*(1+C19)	=MIN(G30,E30*H30)	=G30-I30	=(1+G\$33)^(C\$3-B30)	=J30*K30
31	=H30*(1+C20)	=MIN(G31,E31*H31)	=G31-I31	=(1+G\$33)^(C\$3-B31)	=J31*K31
32	=H31*(1+C21)	=MIN(G32,E32*H32)	=G32-I32	=(1+G\$33)^(C\$3-B32)	=J32*K32
33					=SOMMA(L25:L32)

- **Nuovo piano di ammortamento**

$$D'_N = M$$

$$\prod_{t=N+1}^{2N} K'_t = \frac{D'_s}{N}, \quad \prod_{t=N+1}^{2N} D'_t = D'_{t-1} - C'_t$$

$$\prod_{t=N+1}^{2N} I'_t = D'_{t-1} \bar{i}, \quad \prod_{t=N+1}^{2N} R'_t = C'_t + I'_t$$

	B	C	D	E	F
35	Anni	Capit	Inter	Rate	Resid
44	8				3584.28
45	9	448.04	775.17	1223.21	3136.25
46	10	448.04	678.28	1126.31	2688.21
47	11	448.04	581.38	1029.42	2240.18
48	12	448.04	484.48	932.52	1792.14
49	13	448.04	387.59	835.62	1344.11
50	14	448.04	290.69	738.73	896.07
51	15	448.04	193.79	641.83	448.04
52	16	448.04	96.90	544.93	0.00

	B	C	D	E	F
35	Anni	Capit	Inter	Rate	Resid
44	=B43+1				=L33
45	=B44+1	=F\$44/C\$3	=L33*G\$33	=C45+D45	=F44-C45
46	=B45+1	=F\$44/C\$3	=F45*G\$33	=C46+D46	=F45-C46
47	=B46+1	=F\$44/C\$3	=F46*G\$33	=C47+D47	=F46-C47
48	=B47+1	=F\$44/C\$3	=F47*G\$33	=C48+D48	=F47-C48
49	=B48+1	=F\$44/C\$3	=F48*G\$33	=C49+D49	=F48-C49
50	=B49+1	=F\$44/C\$3	=F49*G\$33	=C50+D50	=F49-C50
51	=B50+1	=F\$44/C\$3	=F50*G\$33	=C51+D51	=F50-C51
52	=B51+1	=F\$44/C\$3	=F51*G\$33	=C52+D52	=F51-C52

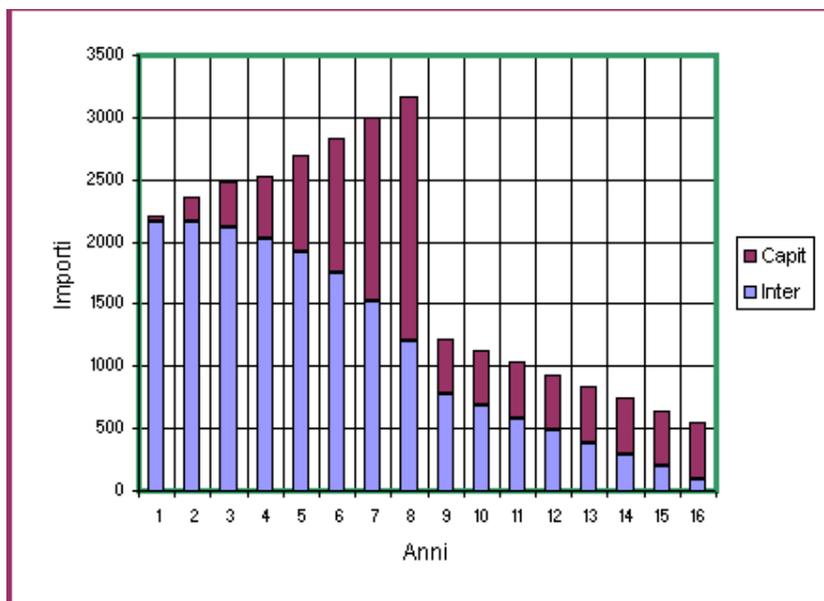
- *Piano di ammortamento a posteriori*

$$\left\{ \begin{array}{l} \prod_{t=1}^N R_t^{\#} = R_t^{pag} \\ \prod_{t=N+1}^{2N} R_t^{\#} = R_t' \end{array} \right. , \quad \prod_{t=1}^{2N} D_t^{\#} = D_{t+1}^{\#} - C_t^{\#} , \quad \prod_{t=1}^{2N} I_t^{\#} = D_t^{\#} \bar{i}$$

$$\prod_{t=1}^{2N} C_t^{\#} = R_t^{\#} - I_t^{\#} , \quad -D_0 + \sum_{t=1}^{2N} R_t^{\#} (1 + \bar{i})^{-t} = 0$$

	B	G	H	I	J
35	Anni	Rate	Inter	Capit	Resid
36	0	-10000.00			10000.00
37	1	2204.25	2162.70	41.55	9958.45
38	2	2362.60	2153.71	208.88	9749.57
39	3	2474.11	2108.54	365.57	9383.99
40	4	2529.21	2029.47	499.74	8884.25
41	5	2697.53	1921.40	776.13	8108.13
42	6	2837.95	1753.54	1084.40	7023.72
43	7	2996.02	1519.02	1477.00	5546.72
44	8	3162.03	1199.59	1962.44	3584.28
45	9	1223.21	775.17	448.04	3136.25
46	10	1126.31	678.28	448.04	2688.21
47	11	1029.42	581.38	448.04	2240.18
48	12	932.52	484.48	448.04	1792.14
49	13	835.62	387.59	448.04	1344.11
50	14	738.73	290.69	448.04	896.07
51	15	641.83	193.79	448.04	448.04
52	16	544.93	96.90	448.04	0.00
53	TIR	21.627%			

	B	G	H	I	J
35	Anni	Rate	Inter	Capit	Resid
36	=C\$2	=F24			=C1
37	=B36+1	=I25	=J36*G\$33	=G37-H37	=J36-I37
38	=B37+1	=I26	=J37*G\$33	=G38-H38	=J37-I38
39	=B38+1	=I27	=J38*G\$33	=G39-H39	=J38-I39
40	=B39+1	=I28	=J39*G\$33	=G40-H40	=J39-I40
41	=B40+1	=I29	=J40*G\$33	=G41-H41	=J40-I41
42	=B41+1	=I30	=J41*G\$33	=G42-H42	=J41-I42
43	=B42+1	=I31	=J42*G\$33	=G43-H43	=J42-I43
44	=B43+1	=I32	=J43*G\$33	=G44-H44	=J43-I44
45	=B44+1	=E45	=J44*G\$33	=G45-H45	=J44-I45
46	=B45+1	=E46	=J45*G\$33	=G46-H46	=J45-I46
47	=B46+1	=E47	=J46*G\$33	=G47-H47	=J46-I47
48	=B47+1	=E48	=J47*G\$33	=G48-H48	=J47-I48
49	=B48+1	=E49	=J48*G\$33	=G49-H49	=J48-I49
50	=B49+1	=E50	=J49*G\$33	=G50-H50	=J49-I50
51	=B50+1	=E51	=J50*G\$33	=G51-H51	=J50-I51
52	=B51+1	=E52	=J51*G\$33	=G52-H52	=J51-I52
53	TIR	=TIR.COST(G36:G52)			



Esercizio E

Un individuo dispone di un capitale D_0 ($=10000$) e al tempo iniziale 0 lo concede in prestito ad un altro soggetto, il quale si impegna a rimborsarlo in N ($=10$) anni, ad un tasso annuo i ($=10\%$), secondo il procedimento di ammortamento francese.

Il creditore, all'atto della riscossione delle rate, deve pagare sulle quote interessi un'imposta caratterizzata da M ($=5$) aliquote progressive a scaglioni e versa il netto ricavo (ossia la rata al netto dell'imposta) in un c/c bancario, caratterizzato da un tasso annuo di interesse pari a i_1 ($=5\%$).

Indicando con

- $A_{h=1,2,\dots,M}$: il vettore delle M aliquote d'imposta,
- $S_{h=1,2,\dots,M-1}$: il vettore delle $M-1$ ampiezze degli scaglioni finiti, considerando l'ultimo scaglione illimitato,

calcolare il saldo del c/c bancario al tempo $N+K$, essendo K ($=5$) un qualsiasi numero naturale.

Si chiede di determinare:

- **il piano di ammortamento originario (del debitore),**
- **il vettore delle imposte annualmente pagate e delle corrispondenti aliquote medie annuali,**
- **il piano di ammortamento netto (del creditore) : piano con quote capitali predeterminate (corrispondenti a quelle dell'ammortamento originario) e tassi di interesse annui (variabili, al netto delle imposte),**
- **il saldo del c/c bancario al tempo finale dell'operazione.**

Nota:

Considerare fissa la grandezza (di tipo durata) N e variabili le altre grandezze (di tipo finanziario) D_0 , i , i_1 , le aliquote d'imposta e le ampiezze degli scaglioni:

$$\begin{array}{l} \begin{array}{l} M \\ \square \\ h=1 \end{array} A_h = (5\% \ 10\% \ 15\% \ 25\% \ 40\%) \\ \begin{array}{l} M-1 \\ \square \\ h=1 \end{array} S_h = (200 \ 100 \ 200 \ 300) \end{array}$$

- **Dati del problema**

	A	B	C	D	E	F	G
1	Capitale	C	10000				
2	Inizio		0				
3	Durata	N	10				
4	Tasso	i	10.00%				
5	Aliquote	A[h]	5.00%	10.00%	15.00%	25.00%	40.00%
6	Scaglioni	S[h]	200	100	200	300	
7	Tasso banca	i1	5.00%				
8	Differimento	K	5				

- **Ammortamento francese relativo al periodo di N anni**

$$\sum_{t=1}^N R_t = \frac{D_0}{a_{\overline{N}|i}}, \quad \sum_{t=1}^N D_t = D_{t-1} - C_t$$

$$\sum_{t=1}^N I_t = D_{t-1} i, \quad \sum_{t=1}^N C_t = R_t - I_t$$

	B	C	D	E	F
10	Anni	Capit	Inter	Rate	Resid
11	0				10000.00
12	1	627.45	1000.00	1627.45	9372.55
13	2	690.20	937.25	1627.45	8682.35
14	3	759.22	868.23	1627.45	7923.13
15	4	835.14	792.31	1627.45	7087.99
16	5	918.66	708.80	1627.45	6169.33
17	6	1010.52	616.93	1627.45	5158.81
18	7	1111.57	515.88	1627.45	4047.24
19	8	1222.73	404.72	1627.45	2824.51
20	9	1345.00	282.45	1627.45	1479.50
21	10	1479.50	147.95	1627.45	0.00

	B	C	D	E	F
10	Anni	Capit	Inter	Rate	Resid
11	=C2				=C1
12	=B11+1	=E12-D12	=F11*C\$4	=F\$11*C\$4/(1-(1+C\$4)^(-C\$3))	=F11-C12
13	=B12+1	=E13-D13	=F12*C\$4	=F\$11*C\$4/(1-(1+C\$4)^(-C\$3))	=F12-C13
14	=B13+1	=E14-D14	=F13*C\$4	=F\$11*C\$4/(1-(1+C\$4)^(-C\$3))	=F13-C14
15	=B14+1	=E15-D15	=F14*C\$4	=F\$11*C\$4/(1-(1+C\$4)^(-C\$3))	=F14-C15
16	=B15+1	=E16-D16	=F15*C\$4	=F\$11*C\$4/(1-(1+C\$4)^(-C\$3))	=F15-C16
17	=B16+1	=E17-D17	=F16*C\$4	=F\$11*C\$4/(1-(1+C\$4)^(-C\$3))	=F16-C17
18	=B17+1	=E18-D18	=F17*C\$4	=F\$11*C\$4/(1-(1+C\$4)^(-C\$3))	=F17-C18
19	=B18+1	=E19-D19	=F18*C\$4	=F\$11*C\$4/(1-(1+C\$4)^(-C\$3))	=F18-C19
20	=B19+1	=E20-D20	=F19*C\$4	=F\$11*C\$4/(1-(1+C\$4)^(-C\$3))	=F19-C20
21	=B20+1	=E21-D21	=F20*C\$4	=F\$11*C\$4/(1-(1+C\$4)^(-C\$3))	=F20-C21

- **Limitazioni (inferiori e superiori) degli scaglioni. Per l'ultimo scaglione si è considerata una limitazione dinamica legata al massimo valore delle quote interessi**

$$L_0 = 0, \quad \prod_{h=1}^{M-1} L_h = L_{h-1} + S_h, \quad L_M = \max \left(L_{M-1}, \prod_{t=1}^N I_t \right)$$

	B	C	D	E	F	G	H
23	Fine		Sca 1	Sca 2	Sca 3	Sca 4	Sca 5
24	Iniz	Sca 1	Sca 2	Sca 3	Sca 4	Sca 5	
25		0	200	300	500	800	1000

	B	C	D	E	F	G	H
23	Fine		Sca 1	Sca 2	Sca 3	Sca 4	Sca 5
24	Iniz	Sca 1	Sca 2	Sca 3	Sca 4	Sca 5	
25		0	=C25+C6	=D25+D6	=E25+E6	=F25+F6	=MAX(G25,D12:D21)

- **Porzioni di quote interessi eccedenti le diverse limitazioni di scaglione**

$$\prod_{t=1}^N \prod_{h=0}^M I_{t,h} = \max(0, I_t - L_h) = I_t - \min(I_t, L_h)$$

- *Imposte relative alle diverse porzioni di quote interessi rientranti nei diversi scaglioni*

$$\prod_{t=1}^N \prod_{h=1}^M P_{t,h} = A_h (I_{t,h-1} - I_{t,h}) = A_h (\min(I_t, L_h) - \min(I_t, L_{h-1}))$$

- *Imposte relative alle diverse quote interessi*

$$\prod_{t=1}^N P_t = \sum_{h=1}^M P_{t,h} = \sum_{h=1}^M A_h (\min(I_t, L_h) - \min(I_t, L_{h-1}))$$

- *Aliquote d'imposta relative alle diverse quote interessi*

$$\prod_{t=1}^N \hat{A}_t = \frac{P_t}{I_t} = \frac{\sum_{h=1}^M A_h (\min(I_t, L_h) - \min(I_t, L_{h-1}))}{I_t} =$$

$$= \sum_{h=1}^M A_h \left(\min\left(1, \frac{L_h}{I_t}\right) - \min\left(1, \frac{L_{h-1}}{I_t}\right) \right)$$

- *Tassi netti annui (variabili, al netto delle imposte)*

$$\prod_{t=1}^N \hat{i}_t = i(1 - \hat{A}_t) = i \left(1 - \sum_{h=1}^M A_h \left(\min\left(1, \frac{L_h}{I_t}\right) - \min\left(1, \frac{L_{h-1}}{I_t}\right) \right) \right)$$

	B	C	D	E	F	G	H	I	J
27	Anni	Sca 1	Sca 2	Sca 3	Sca 4	Sca 5	Imp	% Imp	Tax net
28	1	10.00	10.00	30.00	75.00	80.00	205.00	20.50%	7.950%
29	2	10.00	10.00	30.00	75.00	54.90	179.90	19.19%	8.081%
30	3	10.00	10.00	30.00	75.00	27.29	152.29	17.54%	8.246%
31	4	10.00	10.00	30.00	73.08	0.00	123.08	15.53%	8.447%
32	5	10.00	10.00	30.00	52.20	0.00	102.20	14.42%	8.558%
33	6	10.00	10.00	30.00	29.23	0.00	79.23	12.84%	8.716%
34	7	10.00	10.00	30.00	3.97	0.00	53.97	10.46%	8.954%
35	8	10.00	10.00	15.71	0.00	0.00	35.71	8.82%	9.118%
36	9	10.00	8.25	0.00	0.00	0.00	18.25	6.46%	9.354%
37	10	7.40	0.00	0.00	0.00	0.00	7.40	5.00%	9.500%

	B	C	D
27	Anni	Sca 1	Sca 2
28	=C\$2+1	=C\$5*(MIN(\$D12,D\$25)-MIN(\$D12,C\$25))	=D\$5*(MIN(\$D12,E\$25)-MIN(\$D12,D\$25))
29	=B28+1	=C\$5*(MIN(\$D13,D\$25)-MIN(\$D13,C\$25))	=D\$5*(MIN(\$D13,E\$25)-MIN(\$D13,D\$25))
30	=B29+1	=C\$5*(MIN(\$D14,D\$25)-MIN(\$D14,C\$25))	=D\$5*(MIN(\$D14,E\$25)-MIN(\$D14,D\$25))
31	=B30+1	=C\$5*(MIN(\$D15,D\$25)-MIN(\$D15,C\$25))	=D\$5*(MIN(\$D15,E\$25)-MIN(\$D15,D\$25))
32	=B31+1	=C\$5*(MIN(\$D16,D\$25)-MIN(\$D16,C\$25))	=D\$5*(MIN(\$D16,E\$25)-MIN(\$D16,D\$25))
33	=B32+1	=C\$5*(MIN(\$D17,D\$25)-MIN(\$D17,C\$25))	=D\$5*(MIN(\$D17,E\$25)-MIN(\$D17,D\$25))
34	=B33+1	=C\$5*(MIN(\$D18,D\$25)-MIN(\$D18,C\$25))	=D\$5*(MIN(\$D18,E\$25)-MIN(\$D18,D\$25))
35	=B34+1	=C\$5*(MIN(\$D19,D\$25)-MIN(\$D19,C\$25))	=D\$5*(MIN(\$D19,E\$25)-MIN(\$D19,D\$25))
36	=B35+1	=C\$5*(MIN(\$D20,D\$25)-MIN(\$D20,C\$25))	=D\$5*(MIN(\$D20,E\$25)-MIN(\$D20,D\$25))
37	=B36+1	=C\$5*(MIN(\$D21,D\$25)-MIN(\$D21,C\$25))	=D\$5*(MIN(\$D21,E\$25)-MIN(\$D21,D\$25))

	E	F
27	Sca 3	Sca 4
28	=E\$5*(MIN(\$D12,F\$25)-MIN(\$D12,E\$25))	=F\$5*(MIN(\$D12,G\$25)-MIN(\$D12,F\$25))
29	=E\$5*(MIN(\$D13,F\$25)-MIN(\$D13,E\$25))	=F\$5*(MIN(\$D13,G\$25)-MIN(\$D13,F\$25))
30	=E\$5*(MIN(\$D14,F\$25)-MIN(\$D14,E\$25))	=F\$5*(MIN(\$D14,G\$25)-MIN(\$D14,F\$25))
31	=E\$5*(MIN(\$D15,F\$25)-MIN(\$D15,E\$25))	=F\$5*(MIN(\$D15,G\$25)-MIN(\$D15,F\$25))
32	=E\$5*(MIN(\$D16,F\$25)-MIN(\$D16,E\$25))	=F\$5*(MIN(\$D16,G\$25)-MIN(\$D16,F\$25))
33	=E\$5*(MIN(\$D17,F\$25)-MIN(\$D17,E\$25))	=F\$5*(MIN(\$D17,G\$25)-MIN(\$D17,F\$25))
34	=E\$5*(MIN(\$D18,F\$25)-MIN(\$D18,E\$25))	=F\$5*(MIN(\$D18,G\$25)-MIN(\$D18,F\$25))
35	=E\$5*(MIN(\$D19,F\$25)-MIN(\$D19,E\$25))	=F\$5*(MIN(\$D19,G\$25)-MIN(\$D19,F\$25))
36	=E\$5*(MIN(\$D20,F\$25)-MIN(\$D20,E\$25))	=F\$5*(MIN(\$D20,G\$25)-MIN(\$D20,F\$25))
37	=E\$5*(MIN(\$D21,F\$25)-MIN(\$D21,E\$25))	=F\$5*(MIN(\$D21,G\$25)-MIN(\$D21,F\$25))

	G	H	I	J
27	Sca 5	Imp	% Imp	Tax net
28	=G\$5*(MIN(\$D12,H\$25)-MIN(\$D12,G\$25))	=SOMMA(C28:G28)	=H28/D12	=C\$4*(1-I28)
29	=G\$5*(MIN(\$D13,H\$25)-MIN(\$D13,G\$25))	=SOMMA(C29:G29)	=H29/D13	=C\$4*(1-I29)
30	=G\$5*(MIN(\$D14,H\$25)-MIN(\$D14,G\$25))	=SOMMA(C30:G30)	=H30/D14	=C\$4*(1-I30)
31	=G\$5*(MIN(\$D15,H\$25)-MIN(\$D15,G\$25))	=SOMMA(C31:G31)	=H31/D15	=C\$4*(1-I31)
32	=G\$5*(MIN(\$D16,H\$25)-MIN(\$D16,G\$25))	=SOMMA(C32:G32)	=H32/D16	=C\$4*(1-I32)
33	=G\$5*(MIN(\$D17,H\$25)-MIN(\$D17,G\$25))	=SOMMA(C33:G33)	=H33/D17	=C\$4*(1-I33)
34	=G\$5*(MIN(\$D18,H\$25)-MIN(\$D18,G\$25))	=SOMMA(C34:G34)	=H34/D18	=C\$4*(1-I34)
35	=G\$5*(MIN(\$D19,H\$25)-MIN(\$D19,G\$25))	=SOMMA(C35:G35)	=H35/D19	=C\$4*(1-I35)
36	=G\$5*(MIN(\$D20,H\$25)-MIN(\$D20,G\$25))	=SOMMA(C36:G36)	=H36/D20	=C\$4*(1-I36)
37	=G\$5*(MIN(\$D21,H\$25)-MIN(\$D21,G\$25))	=SOMMA(C37:G37)	=H37/D21	=C\$4*(1-I37)

- *il piano di ammortamento netto (del creditore) e il saldo del c/c bancario al tempo finale dell'operazione*

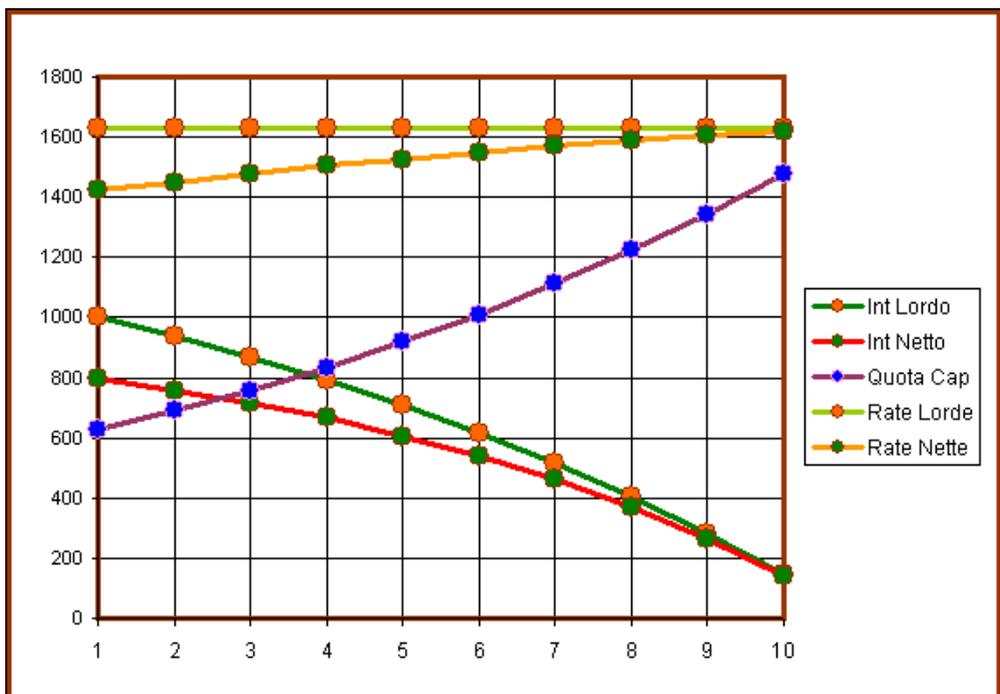
$$\prod_{t=1}^N C_t, \quad \prod_{t=1}^N D_t = D_{t-1} - C_t$$

$$\prod_{t=1}^N \hat{i}_t = D_{t-1} \hat{i}_t, \quad \prod_{t=1}^N \hat{R}_t = C_t + \hat{i}_t$$

$$M = \sum_{t=1}^N \hat{R}_t (1+i_1)^{N+k-t}$$

	B	C	D	E	F	G	H
39	Anni	Capit	Inter	Rate	Resid	Coeff.	Mont
40	0				10000.00		
41	1	627.45	795.00	1422.45	9372.55	1.979932	2816.36
42	2	690.20	757.35	1447.55	8682.35	1.885649	2729.58
43	3	759.22	715.94	1475.16	7923.13	1.795856	2649.18
44	4	835.14	669.23	1504.38	7087.99	1.710339	2572.99
45	5	918.66	606.60	1525.25	6169.33	1.628895	2484.48
46	6	1010.52	537.70	1548.22	5158.81	1.551328	2401.80
47	7	1111.57	461.91	1573.48	4047.24	1.477455	2324.75
48	8	1222.73	369.02	1591.75	2824.51	1.407100	2239.75
49	9	1345.00	264.21	1609.21	1479.50	1.340096	2156.49
50	10	1479.50	140.55	1620.06	0.00	1.276282	2067.65
51							24443.02

	B	C	D	E	F	G	H
39	Anni	Capit	Inter	Rate	Resid	Coeff.	Mont
40	=J31				=C1		
41	=C2+1	=C12	=F40*J28	=C41+D41	=F40-C41	=(1+C\$7)^(C\$3+C\$8-B41)	=E41*G41
42	=B41+1	=C13	=F41*J29	=C42+D42	=F41-C42	=(1+C\$7)^(C\$3+C\$8-B42)	=E42*G42
43	=B42+1	=C14	=F42*J30	=C43+D43	=F42-C43	=(1+C\$7)^(C\$3+C\$8-B43)	=E43*G43
44	=B43+1	=C15	=F43*J31	=C44+D44	=F43-C44	=(1+C\$7)^(C\$3+C\$8-B44)	=E44*G44
45	=B44+1	=C16	=F44*J32	=C45+D45	=F44-C45	=(1+C\$7)^(C\$3+C\$8-B45)	=E45*G45
46	=B45+1	=C17	=F45*J33	=C46+D46	=F45-C46	=(1+C\$7)^(C\$3+C\$8-B46)	=E46*G46
47	=B46+1	=C18	=F46*J34	=C47+D47	=F46-C47	=(1+C\$7)^(C\$3+C\$8-B47)	=E47*G47
48	=B47+1	=C19	=F47*J35	=C48+D48	=F47-C48	=(1+C\$7)^(C\$3+C\$8-B48)	=E48*G48
49	=B48+1	=C20	=F48*J36	=C49+D49	=F48-C49	=(1+C\$7)^(C\$3+C\$8-B49)	=E49*G49
50	=B49+1	=C21	=F49*J37	=C50+D50	=F49-C50	=(1+C\$7)^(C\$3+C\$8-B50)	=E50*G50
51							=SOMMA(H41:H50)



Esercizio F

Al tempo iniziale 0 un individuo contrae un mutuo di ammontare D_0 (=10000) con obbligo di rimborso secondo la metodologia dell'ammortamento con rate in progressione geometrica di

ragione $1+g$ ($1+15\%$) in S ($=12$) anni al tasso annuo d'interesse i ($=4\%$). Dopo S_1 ($=7$) pagamenti l'individuo accetta la modifica in ammortamento con quote capitali in progressione geometrica, sempre di ragione $1+g$, per i successivi $S-S_1$ anni e al tasso annuo d'interesse i_2 ($=8\%$).

All'atto della riscossione delle rate, il creditore deve versare all'erario un'imposta di aliquota $A\%$ ($=50\%$) sulla quota interesse, limitatamente alla quota eccedente la franchigia F ($=350$) e versa il netto ricavo annuale in un c/c bancario caratterizzato da un tasso annuo di interesse i_3 (2%).

Si chiede di determinare:

- **il piano di ammortamento originario,**
 - **il piano di ammortamento modificato,**
- e, con riferimento all'ammortamento modificato,**
- **le imposte pagate dal creditore sulle quote interesse,**
 - **i saldi annuali del conto corrente bancario sino al termine dell'ammortamento,**
 - **la nuda proprietà, l'usufrutto, il valore del prestito e l'indice del grado di capitalizzazione al tempo 0 , sia dalla parte del debitore che da quella del creditore, a un tasso annuo d'interesse pari alla media dei due tassi originari.**

Nota:

Considerare fisse la grandezza (di tipo durata) S ed S_1 e variabili le altre grandezze (di tipo finanziario) D_0 , g , i , i_2 , A , F e i_3 .

	A	B	C
1	Debito	D_0	10000
2	Durata	S	12
3	Ragione	g	15.00%
4	Modifica	S_1	7
5	Tasso	i	4.00%
6	Tasso 2	i_2	8.00%
7	Aliquota	A	50.00%
8	Franchigia	F	350
9	Tasso 3	i_3	2.00%

- Piano di ammortamento con rate variabili in progressione geometrica (francese, se $g=0$) relativo al periodo di S anni

$$D_0 = \sum_{t=1}^S R_t (1+i)^{-t} = R_1 \sum_{t=1}^S (1+g)^{t-1} (1+i)^{-t} = R_1 (Ga)_{\overline{S}|i}^{(g)} \Rightarrow R_1 = \frac{D_0}{(Ga)_{\overline{S}|i}^{(g)}}$$

$$\prod_{t=1}^N R_t = \frac{D_0 (1+g)^{t-1}}{(Ga)_{\overline{S}|i}^{(g)}} = \begin{cases} 0 = g \Rightarrow \frac{D_0}{a_{\overline{S}|i}} = \frac{D_0}{\frac{1-(1+i)^{-S}}{i}} \\ 0 \neq g = i \Rightarrow \frac{D_0 (1+g)^{t-1}}{S} = \frac{D_0 (1+g)^t}{S} \\ 0 \neq g \neq i \Rightarrow \frac{D_0 (1+g)^{t-1}}{\frac{a_{\overline{S}|i-g}}{1+g}} = \frac{D_0 (1+g)^{t-1}}{1 - \left(1 + \frac{i-g}{1+g}\right)^{-S}} \cdot \frac{1+g}{i-g} \end{cases}$$

$$\prod_{t=1}^N D_t = D_{t-1} - C_t, \quad \prod_{t=1}^N I_t = D_{t-1} i, \quad \prod_{t=1}^N C_t = R_t - I_t$$

	A	B	C
12	$(i-g)/(1+g)$		-9.5652%
13	$Ga[S]i$		21.288640

	A	B	C
12	$(i-g)/(1+g)$		$=(C5-C3)/(1+C3)$
13	$Ga[S]j$		$=SE(C5=C3,C2/(1+C3),(1-(1+C12)^{-C2})/(C5-C3))$

	E	F	G	H	I
1	Anni	Capit	Inter	Rate	Residuo
2	0				10000.00
3	1	69.73	400.00	469.73	9930.27
4	2	142.98	397.21	540.19	9787.28
5	3	229.73	391.49	621.22	9557.55
6	4	332.10	382.30	714.41	9225.45
7	5	452.55	369.02	821.57	8772.90
8	6	593.89	350.92	944.80	8179.01
9	7	759.36	327.16	1086.52	7419.65
10	8	952.72	296.79	1249.50	6466.93
11	9	1178.25	258.68	1436.93	5288.68
12	10	1440.92	211.55	1652.47	3847.76
13	11	1746.43	153.91	1900.34	2101.33
14	12	2101.33	84.05	2185.39	0.00

	E	F	G	H	I
1	Anni	Capit	Inter	Rate	Residuo
2	0				=C1
3	1	=H3-G3	=I2*C\$5	=I\$2*(1+C\$3)^(E3-1)/C\$13	=I2-F3
4	2	=H4-G4	=I3*C\$5	=I\$2*(1+C\$3)^(E4-1)/C\$13	=I3-F4
5	3	=H5-G5	=I4*C\$5	=I\$2*(1+C\$3)^(E5-1)/C\$13	=I4-F5
6	4	=H6-G6	=I5*C\$5	=I\$2*(1+C\$3)^(E6-1)/C\$13	=I5-F6
7	5	=H7-G7	=I6*C\$5	=I\$2*(1+C\$3)^(E7-1)/C\$13	=I6-F7
8	6	=H8-G8	=I7*C\$5	=I\$2*(1+C\$3)^(E8-1)/C\$13	=I7-F8
9	7	=H9-G9	=I8*C\$5	=I\$2*(1+C\$3)^(E9-1)/C\$13	=I8-F9
10	8	=H10-G10	=I9*C\$5	=I\$2*(1+C\$3)^(E10-1)/C\$13	=I9-F10
11	9	=H11-G11	=I10*C\$5	=I\$2*(1+C\$3)^(E11-1)/C\$13	=I10-F11
12	10	=H12-G12	=I11*C\$5	=I\$2*(1+C\$3)^(E12-1)/C\$13	=I11-F12
13	11	=H13-G13	=I12*C\$5	=I\$2*(1+C\$3)^(E13-1)/C\$13	=I12-F13
14	12	=H14-G14	=I13*C\$5	=I\$2*(1+C\$3)^(E14-1)/C\$13	=I13-F14

- **Piano di ammortamento modificato con quota capitale in progressione geometrica (italiano, se $g=0$) relativo al periodo di $S-S_1$ anni**

$$D_{S_1} = \sum_{t=S_1+1}^S C_t = C_{S_1+1} \sum_{t=S_1+1}^S (1+g)^{t-S_1-1} = C_{S_1+1} s_{\overline{S-S_1}|g} \Rightarrow C_{S_1+1} = \frac{D_{S_1}}{s_{\overline{S-S_1}|g}}$$

$$\left\{ \begin{array}{l} \sum_{t=1}^{S_1} \hat{C}_t = C_t \\ \sum_{t=S_1+1}^S \hat{C}_t = \frac{D_{S_1} (1+g)^{t-S_1-1}}{s_{\overline{S-S_1}|g}} = \begin{cases} g=0 \Rightarrow = \frac{D_{S_1} (1+g)^{t-S_1-1}}{S-S_1} \\ g \neq 0 \Rightarrow = \frac{D_{S_1} (1+g)^{t-S_1-1}}{(1+g)^{S-S_1} - 1} g \end{cases} \\ \sum_{t=1}^{S_1} \hat{D}_t = D_t \\ \sum_{t=S_1+1}^S \hat{D}_t = \hat{D}_{t-1} - \hat{C}_t \\ \sum_{t=1}^{S_1} \hat{I}_t = I_t \\ \sum_{t=S_1+1}^S \hat{I}_t = \hat{D}_{t-1} i_2 \\ \sum_{t=1}^{S_1} \hat{R}_t = R_t \\ \sum_{t=S_1+1}^S \hat{R}_t = \hat{C}_t + \hat{I}_t \end{array} \right.$$

	A	B	C
14	$s[\overline{S-S_1} g]$		6.742381

	A	B	C
14	$s[\overline{S-S_1} g]$		$=SE(C3=0,C2-C4,((1+C3)^(C2-C4)-1)/C3)$

	E	F	G	H	I
18	Anni	Capit	Inter	Rate	Residuo
19	0				10000.00
20	1	69.73	400.00	469.73	9930.27
21	2	142.98	397.21	540.19	9787.28
22	3	229.73	391.49	621.22	9557.55
23	4	332.10	382.30	714.41	9225.45
24	5	452.55	369.02	821.57	8772.90
25	6	593.89	350.92	944.80	8179.01
26	7	759.36	327.16	1086.52	7419.65
27	8	1100.45	593.57	1694.02	6319.20
28	9	1265.52	505.54	1771.05	5053.68
29	10	1455.34	404.29	1859.64	3598.34
30	11	1673.65	287.87	1961.51	1924.69
31	12	1924.69	153.98	2078.67	0.00

	E	F	G	H	I
18	Anni	Capit	Inter	Rate	Residuo
19	0				=I2
20	1	=F3	=G3	=H3	=I3
21	2	=F4	=G4	=H4	=I4
22	3	=F5	=G5	=H5	=I5
23	4	=F6	=G6	=H6	=I6
24	5	=F7	=G7	=H7	=I7
25	6	=F8	=G8	=H8	=I8
26	7	=F9	=G9	=H9	=I9
27	8	=I\$26*(1+C\$3)^(E27-C\$4-1)/C\$14	=I26*C\$6	=F27+G27	=I26-F27
28	9	=I\$26*(1+C\$3)^(E28-C\$4-1)/C\$14	=I27*C\$6	=F28+G28	=I27-F28
29	10	=I\$26*(1+C\$3)^(E29-C\$4-1)/C\$14	=I28*C\$6	=F29+G29	=I28-F29
30	11	=I\$26*(1+C\$3)^(E30-C\$4-1)/C\$14	=I29*C\$6	=F30+G30	=I29-F30
31	12	=I\$26*(1+C\$3)^(E31-C\$4-1)/C\$14	=I30*C\$6	=F31+G31	=I30-F31

- *Imposte pagate dal creditore sulle quote interesse e saldi annuali del conto corrente bancario*

$$\begin{aligned} \prod_{t=1}^S Imp_t &= \max((\hat{I}_t - F), 0) A \\ \prod_{t=1}^S \hat{I}_t^{\#} &= \hat{I}_t - Imp_t \\ \prod_{t=1}^S \hat{R}_t^{\#} &= \hat{R}_t - Imp_t \\ Z_0 &= 0, \quad \prod_{t=1}^S Z_t = \hat{R}_t^{\#} + Z_{t-1}(1 + i_3) \end{aligned}$$

	K	L	M	N
18	Imposta	Inter2	Rate2	Saldo
19				0
20	25.00	375.00	444.73	444.73
21	23.61	373.61	516.59	970.22
22	20.75	370.75	600.48	1590.10
23	16.15	366.15	698.26	2320.16
24	9.51	359.51	812.06	3178.62
25	0.46	350.46	944.35	4186.54
26	0.00	327.16	1086.52	5356.79
27	121.79	471.79	1572.23	7036.16
28	77.77	427.77	1693.28	8870.17
29	27.15	377.15	1832.49	10880.06
30	0.00	287.87	1961.51	13059.18
31	0.00	153.98	2078.67	15399.03

	K	L	M	N
18	Imposta	Inter2	Rate2	Saldo
19				0
20	=MAX((G20-C\$8),0)*C\$7	=G20-K20	=H20-K20	=M20+N19*(1+C\$9)
21	=MAX((G21-C\$8),0)*C\$7	=G21-K21	=H21-K21	=M21+N20*(1+C\$9)
22	=MAX((G22-C\$8),0)*C\$7	=G22-K22	=H22-K22	=M22+N21*(1+C\$9)
23	=MAX((G23-C\$8),0)*C\$7	=G23-K23	=H23-K23	=M23+N22*(1+C\$9)
24	=MAX((G24-C\$8),0)*C\$7	=G24-K24	=H24-K24	=M24+N23*(1+C\$9)
25	=MAX((G25-C\$8),0)*C\$7	=G25-K25	=H25-K25	=M25+N24*(1+C\$9)
26	=MAX((G26-C\$8),0)*C\$7	=G26-K26	=H26-K26	=M26+N25*(1+C\$9)
27	=MAX((G27-C\$8),0)*C\$7	=G27-K27	=H27-K27	=M27+N26*(1+C\$9)
28	=MAX((G28-C\$8),0)*C\$7	=G28-K28	=H28-K28	=M28+N27*(1+C\$9)
29	=MAX((G29-C\$8),0)*C\$7	=G29-K29	=H29-K29	=M29+N28*(1+C\$9)
30	=MAX((G30-C\$8),0)*C\$7	=G30-K30	=H30-K30	=M30+N29*(1+C\$9)
31	=MAX((G31-C\$8),0)*C\$7	=G31-K31	=H31-K31	=M31+N30*(1+C\$9)

- *la nuda proprietà, l'usufrutto, il valore del prestito e l'indice del grado di capitalizzazione al tempo 0, sia dalla parte del debitore che da quella del creditore*

$$i_m = \frac{i + i_2}{2}$$

$$NPr = \sum_{t=1}^s \hat{C}_t (1 + i_m)^{-t}$$

$$Usu = \sum_{t=1}^s \hat{I}_t (1 + i_m)^{-t}, \quad Val = \sum_{t=1}^s \hat{R}_t (1 + i_m)^{-t}$$

$$Usu^* = \sum_{t=1}^s \hat{I}_t^* (1 + i_m)^{-t} = Usu - \sum_{t=1}^s Imp_t (1 + i_m)^{-t} = Usu - Imp$$

$$Val^* = \sum_{t=1}^s \hat{R}_t^* (1 + i_m)^{-t} = Val - \sum_{t=1}^s Imp_t (1 + i_m)^{-t} = Val - Imp$$

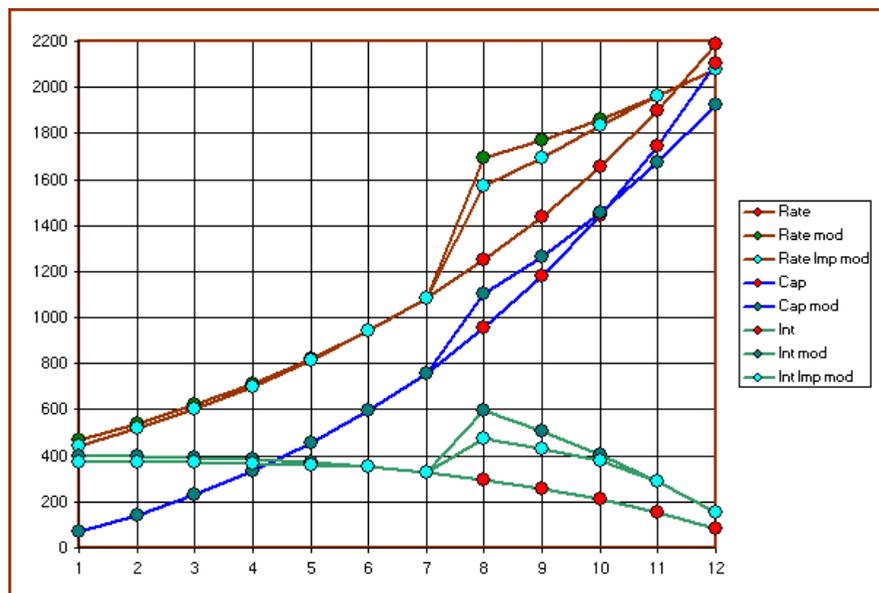
$$GrC = \frac{Usu}{Val} = \frac{\sum_{t=1}^s \hat{I}_t (1+i_m)^{-t}}{\sum_{t=1}^s \hat{R}_t (1+i_m)^{-t}}$$

$$GrC^{\#} = \frac{Usu^{\#}}{Val^{\#}} = \frac{\sum_{t=1}^s \hat{I}_t^{\#} (1+i_m)^{-t}}{\sum_{t=1}^s \hat{R}_t^{\#} (1+i_m)^{-t}} = \frac{Usu - Imp}{Val - Imp} = GrC - \frac{Imp \cdot Npr}{Val \cdot Val^{\#}}$$

	P	Q	R	S	T	U	V
18	Coeff	NPro	Usu	Val	Imp	Usu2	Val2
19							
20	0.9434	65.79	377.36	443.15	23.58	353.77	419.56
21	0.8900	127.25	353.52	480.77	21.01	332.51	459.76
22	0.8396	192.89	328.70	521.59	17.42	311.29	504.17
23	0.7921	263.06	302.82	565.88	12.79	290.03	553.08
24	0.7473	338.17	275.75	613.92	7.11	268.65	606.82
25	0.7050	418.67	247.38	666.05	0.32	247.06	665.73
26	0.6651	505.02	217.58	722.60	0.00	217.58	722.60
27	0.6274	690.44	372.41	1062.85	76.41	296.00	986.44
28	0.5919	749.06	299.23	1048.28	46.03	253.20	1002.25
29	0.5584	812.66	225.76	1038.41	15.16	210.60	1023.25
30	0.5268	881.66	151.64	1033.30	0.00	151.64	1033.30
31	0.4970	956.51	76.52	1033.03	0.00	76.52	1033.03
32		6001.16	3228.67	9229.83	219.83	3008.84	9010.00
33			Grado	34.98%		Grado	33.39%

	P	Q	R	S
	<i>Coeff</i>	<i>NPro</i>	<i>Usu</i>	<i>Val</i>
18				
19				
20	$= (1 + (C\$5 + C\$6) / 2)^{-E20}$	$= F20 * \$P20$	$= G20 * \$P20$	$= H20 * \$P20$
21	$= (1 + (C\$5 + C\$6) / 2)^{-E21}$	$= F21 * \$P21$	$= G21 * \$P21$	$= H21 * \$P21$
22	$= (1 + (C\$5 + C\$6) / 2)^{-E22}$	$= F22 * \$P22$	$= G22 * \$P22$	$= H22 * \$P22$
23	$= (1 + (C\$5 + C\$6) / 2)^{-E23}$	$= F23 * \$P23$	$= G23 * \$P23$	$= H23 * \$P23$
24	$= (1 + (C\$5 + C\$6) / 2)^{-E24}$	$= F24 * \$P24$	$= G24 * \$P24$	$= H24 * \$P24$
25	$= (1 + (C\$5 + C\$6) / 2)^{-E25}$	$= F25 * \$P25$	$= G25 * \$P25$	$= H25 * \$P25$
26	$= (1 + (C\$5 + C\$6) / 2)^{-E26}$	$= F26 * \$P26$	$= G26 * \$P26$	$= H26 * \$P26$
27	$= (1 + (C\$5 + C\$6) / 2)^{-E27}$	$= F27 * \$P27$	$= G27 * \$P27$	$= H27 * \$P27$
28	$= (1 + (C\$5 + C\$6) / 2)^{-E28}$	$= F28 * \$P28$	$= G28 * \$P28$	$= H28 * \$P28$
29	$= (1 + (C\$5 + C\$6) / 2)^{-E29}$	$= F29 * \$P29$	$= G29 * \$P29$	$= H29 * \$P29$
30	$= (1 + (C\$5 + C\$6) / 2)^{-E30}$	$= F30 * \$P30$	$= G30 * \$P30$	$= H30 * \$P30$
31	$= (1 + (C\$5 + C\$6) / 2)^{-E31}$	$= F31 * \$P31$	$= G31 * \$P31$	$= H31 * \$P31$
32		$= SOMMA(Q20:Q31)$	$= SOMMA(R20:R31)$	$= SOMMA(S20:S31)$
33			<i>Grado</i>	$= R32/S32$

	P	T	U	V
	<i>Coeff</i>	<i>Imp</i>	<i>Usu2</i>	<i>Val2</i>
18				
19				
20	$= (1 + (C\$5 + C\$6) / 2)^{-E20}$	$= K20 * \$P20$	$= L20 * \$P20$	$= M20 * \$P20$
31	$= (1 + (C\$5 + C\$6) / 2)^{-E31}$	$= K31 * \$P31$	$= L31 * \$P31$	$= M31 * \$P31$
32		$= SOMMA(T20:T31)$	$= SOMMA(U20:U31)$	$= SOMMA(V20:V31)$
33			<i>Grado</i>	$= U32/V32$



Esercizio G

Al tempo iniziale 0 un soggetto contrae un mutuo avente le seguenti caratteristiche:

- *Importo: D_0 (=100000)*
- *Durata del prestito: T_1 (=10 anni)*
- *Tasso annuo di interesse i_1 (=6%)*
- *Metodologia: ammortamento progressivo con quote capitale in progressione aritmetica di ragione pari a G (= -100),*

ma si accorge di non poter impiegare il capitale ottenuto D_0 , per cui, dopo w (=1) ann1 di improduttività, decide di darlo in prestito, concedendo un mutuo alle seguenti condizioni:

- *Durata del prestito: T_2 (=6 anni oppure =12 anni)*
- *Tasso annuo di interesse: i_2 (=9%)*
- *Metodologia: ammortamento di tipo francese,*

versando oppure prelevando le differenze in oppure da un c/c bancario caratterizzato dai due tassi annui di interesse, rispettivamente creditore e debitore i_3 (=2%) e i_4 (=12%).

Si chiede di determinare:

- *i diversi piani di ammortamento,*
- *il saldo del c/c bancario al termine di entrambe le operazioni di mutuo, nelle due ipotesi di durata del secondo mutuo,*
- *supponendo di poter distribuire il secondo ammortamento tra due possibili durate T_2 , secondo una percentuale A_h % di ammortamento secondo la durata breve e una percentuale $(1-A_h)$ % di ammortamento secondo la durata lunga, indicare per quale percentuale A_h % si ottiene il massimo valore del saldo del c/c bancario alla fine dell'operazione.*

Nota:

Considerare fisse la grandezza (di tipo durata) T_1 , T_2 e w e variabili le altre grandezze (di tipo finanziario) D_0 , G , i , i_2 , i_3 , i_4 ed A_h

$$\prod_{h=0}^{20} A_h = \frac{h}{100} = (0\% \ 5\% \ 10\% \ \dots \ 90\% \ 95\% \ 100\%)$$

- Dati del problema**

	A	B	C	D	E
1	D_0	100000			
2	G	-100			
3	i_1	6.00%	T_1	10	
4	i_2	9.00%	T_2	6	12
5	i_3	2.00%			
6	i_4	12.00%			
7	Inizio	0			
8	Reimpiego	1			
9	Passo A%	5%			

- Piano di ammortamento con quote capitale in progressione aritmetica (italiano, se $G=0$) relativo al periodo di T_1 anni**

$$D_0 = \sum_{t=1}^{T_1} C_t = \sum_{t=1}^{T_1} (C_1 + G(t-1)) = C_1 T_1 + G \sum_{t=1}^{T_1} (t-1) = C_1 T_1 + \frac{G(T_1-1)T_1}{2}$$

$$\Rightarrow C_1 = \frac{D_0}{T_1} - \frac{G(T_1-1)}{2}$$

$$\prod_{t=1}^N C_t = \frac{D_0}{T_1} - \frac{G(T_1-1)}{2} + G(t-1) = \frac{D_0}{T_1} - \frac{G(2t-T_1-1)}{2}$$

$$\prod_{t=1}^N D_t = D_{t-1} - C_t, \quad \prod_{t=1}^N I_t = D_{t-1} i_t, \quad \prod_{t=1}^N R_t = C_t + I_t$$

	B	C	D	E	F
11	Tempi	Capit	Inter	Rata	Resid
12	0				100000
13	1	10450	6000	16450	89550
14	2	10350	5373	15723	79200
15	3	10250	4752	15002	68950
16	4	10150	4137	14287	58800
17	5	10050	3528	13578	48750
18	6	9950	2925	12875	38800
19	7	9850	2328	12178	28950
20	8	9750	1737	11487	19200
21	9	9650	1152	10802	9550
22	10	9550	573	10123	0
23	11			0	
24	12			0	
25	13			0	

	B	C	D	E	F
11	Tempi	Capit	Inter	Rata	Resid
12	=B7				=B1
13	=B12+1	=B\$1/D\$3+B\$2*(2*B13-D\$3-1)/2	=F12*B\$3	=C13+D13	=F12-C13
14	=B13+1	=B\$1/D\$3+B\$2*(2*B14-D\$3-1)/2	=F13*B\$3	=C14+D14	=F13-C14
15	=B14+1	=B\$1/D\$3+B\$2*(2*B15-D\$3-1)/2	=F14*B\$3	=C15+D15	=F14-C15
16	=B15+1	=B\$1/D\$3+B\$2*(2*B16-D\$3-1)/2	=F15*B\$3	=C16+D16	=F15-C16
17	=B16+1	=B\$1/D\$3+B\$2*(2*B17-D\$3-1)/2	=F16*B\$3	=C17+D17	=F16-C17
18	=B17+1	=B\$1/D\$3+B\$2*(2*B18-D\$3-1)/2	=F17*B\$3	=C18+D18	=F17-C18
19	=B18+1	=B\$1/D\$3+B\$2*(2*B19-D\$3-1)/2	=F18*B\$3	=C19+D19	=F18-C19
20	=B19+1	=B\$1/D\$3+B\$2*(2*B20-D\$3-1)/2	=F19*B\$3	=C20+D20	=F19-C20
21	=B20+1	=B\$1/D\$3+B\$2*(2*B21-D\$3-1)/2	=F20*B\$3	=C21+D21	=F20-C21
22	=B21+1	=B\$1/D\$3+B\$2*(2*B22-D\$3-1)/2	=F21*B\$3	=C22+D22	=F21-C22
23	=B22+1			0	
24	=B23+1			0	
25	=B24+1			0	

- **Piano di ammortamento di tipo francese al periodo di T_2 anni e saldo del C/C bancario (nelle due ipotesi di durata del secondo mutuo)**

$$\prod_{k=1}^2 \prod_{t=w+1}^{T_{2,k}+w} R_t^{(k)} = \frac{D_0}{a_{\overline{T_{2,k}}|i_2}}, \quad \prod_{k=1}^2 \prod_{t=w+1}^{T_{2,k}+w} D_t^{(k)} = D_{t-1}^{(k)} - C_t^{(k)}$$

$$\prod_{k=1}^2 \prod_{t=w+1}^{T_{2,k}+w} I_t^{(k)} = D_{t-1}^{(k)} i_2, \quad \prod_{k=1}^2 \prod_{t=w+1}^{T_{2,k}+w} C_t^{(k)} = R_t^{(k)} - I_t^{(k)}$$

$$\prod_{k=1}^2 \prod_{t=w+1}^{T_{2,k}+w} Cf_t^{(k)} = R_t^{(k)} - R_t$$

$$\prod_{k=1}^2 S_0^{(k)} = 0, \quad \prod_{k=1}^2 \prod_{t=w+1}^{T_{2,k}+w} S_t^{(k)} = S_{t-1}^{(k)} \cdot \begin{cases} (1+i_3) & \text{se } S_{t-1}^{(k)} \geq 0 \\ (1+i_4) & \text{se } S_{t-1}^{(k)} < 0 \end{cases} + Cf_t^{(k)}$$

	B	C	D	E	F	G	H	I	J
27	Tempi	Capit	Inter	Rata	Resid		T	Cflow	Saldi
28	0						0		0
29	1				100000		1	-16450	-16450
30	2	13292	9000	22292	86708		2	6569	-11855
31	3	14488	7804	22292	72220		3	7290	-5988
32	4	15792	6500	22292	56428		4	8005	1299
33	5	17213	5078	22292	39214		5	8714	10039
34	6	18763	3529	22292	20451		6	9417	19657
35	7	20451	1841	22292	0		7	10114	30164
36	8			0			8	-11487	19280
37	9			0			9	-10802	8864
38	10			0			10	-10123	-1082
39	11			0			11	0	-1212
40	12			0			12	0	-1358
41	13			0			13	0	-1520

	B	C	D	E	F
27	Tempi	Capit	Inter	Rata	Resid
28	0				
29	=B8				=B1
30	=B29+1	=E30-D30	=F29*B\$4	=F\$29*B\$4/(1-(1+B\$4)^-D\$4)	=F29-C30
31	=B30+1	=E31-D31	=F30*B\$4	=F\$29*B\$4/(1-(1+B\$4)^-D\$4)	=F30-C31
32	=B31+1	=E32-D32	=F31*B\$4	=F\$29*B\$4/(1-(1+B\$4)^-D\$4)	=F31-C32
33	=B32+1	=E33-D33	=F32*B\$4	=F\$29*B\$4/(1-(1+B\$4)^-D\$4)	=F32-C33
34	=B33+1	=E34-D34	=F33*B\$4	=F\$29*B\$4/(1-(1+B\$4)^-D\$4)	=F33-C34
35	=B34+1	=E35-D35	=F34*B\$4	=F\$29*B\$4/(1-(1+B\$4)^-D\$4)	=F34-C35
36	=B35+1			0	
37	=B36+1			0	
38	=B37+1			0	
39	=B38+1			0	
40	=B39+1			0	
41	=B40+1			0	

	H	I	J
27	T	Cflow	Saldi
28	=B28		0
29	=B29	=E29-E13	=J28*((1+B\$5)*(J28>=0)+(1+B\$6)*(J28<0))+I29
30	=B30	=E30-E14	=J29*((1+B\$5)*(J29>=0)+(1+B\$6)*(J29<0))+I30
31	=B31	=E31-E15	=J30*((1+B\$5)*(J30>=0)+(1+B\$6)*(J30<0))+I31
32	=B32	=E32-E16	=J31*((1+B\$5)*(J31>=0)+(1+B\$6)*(J31<0))+I32
33	=B33	=E33-E17	=J32*((1+B\$5)*(J32>=0)+(1+B\$6)*(J32<0))+I33
34	=B34	=E34-E18	=J33*((1+B\$5)*(J33>=0)+(1+B\$6)*(J33<0))+I34
35	=B35	=E35-E19	=J34*((1+B\$5)*(J34>=0)+(1+B\$6)*(J34<0))+I35
36	=B36	=E36-E20	=J35*((1+B\$5)*(J35>=0)+(1+B\$6)*(J35<0))+I36
37	=B37	=E37-E21	=J36*((1+B\$5)*(J36>=0)+(1+B\$6)*(J36<0))+I37
38	=B38	=E38-E22	=J37*((1+B\$5)*(J37>=0)+(1+B\$6)*(J37<0))+I38
39	=B39	=E39-E23	=J38*((1+B\$5)*(J38>=0)+(1+B\$6)*(J38<0))+I39
40	=B40	=E40-E24	=J39*((1+B\$5)*(J39>=0)+(1+B\$6)*(J39<0))+I40
41	=B41	=E41-E25	=J40*((1+B\$5)*(J40>=0)+(1+B\$6)*(J40<0))+I41

	B	C	D	E	F	G	H	I	J
43	Tempi	Capit	Inter	Rata	Resid		T	Cflow	Saldi
44	0						0		0
45	1				100000		1	-16450	-16450
46	2	4965	9000	13965	95035		2	-1758	-20182
47	3	5412	8553	13965	89623		3	-1037	-23641
48	4	5899	8066	13965	83724		4	-322	-26800
49	5	6430	7535	13965	77294		5	387	-29628
50	6	7009	6956	13965	70286		6	1090	-32094
51	7	7639	6326	13965	62646		7	1787	-34158
52	8	8327	5638	13965	54319		8	2478	-35779
53	9	9076	4889	13965	45243		9	3163	-36909
54	10	9893	4072	13965	35350		10	3842	-37496
55	11	10784	3181	13965	24566		11	13965	-28031
56	12	11754	2211	13965	12812		12	13965	-17429
57	13	12812	1153	13965	0		13	13965	-5556

	B	C	D	E	F
43	Tempi	Capit	Inter	Rata	Resid
44	0				
45	=B8				=B1
46	=B45+1	=E46-D46	=F45*B\$4	=F\$45*B\$4/(1-(1+B\$4)^-E\$4)	=F45-C46
47	=B46+1	=E47-D47	=F46*B\$4	=F\$45*B\$4/(1-(1+B\$4)^-E\$4)	=F46-C47
48	=B47+1	=E48-D48	=F47*B\$4	=F\$45*B\$4/(1-(1+B\$4)^-E\$4)	=F47-C48
49	=B48+1	=E49-D49	=F48*B\$4	=F\$45*B\$4/(1-(1+B\$4)^-E\$4)	=F48-C49
50	=B49+1	=E50-D50	=F49*B\$4	=F\$45*B\$4/(1-(1+B\$4)^-E\$4)	=F49-C50
51	=B50+1	=E51-D51	=F50*B\$4	=F\$45*B\$4/(1-(1+B\$4)^-E\$4)	=F50-C51
52	=B51+1	=E52-D52	=F51*B\$4	=F\$45*B\$4/(1-(1+B\$4)^-E\$4)	=F51-C52
53	=B52+1	=E53-D53	=F52*B\$4	=F\$45*B\$4/(1-(1+B\$4)^-E\$4)	=F52-C53
54	=B53+1	=E54-D54	=F53*B\$4	=F\$45*B\$4/(1-(1+B\$4)^-E\$4)	=F53-C54
55	=B54+1	=E55-D55	=F54*B\$4	=F\$45*B\$4/(1-(1+B\$4)^-E\$4)	=F54-C55
56	=B55+1	=E56-D56	=F55*B\$4	=F\$45*B\$4/(1-(1+B\$4)^-E\$4)	=F55-C56
57	=B56+1	=E57-D57	=F56*B\$4	=F\$45*B\$4/(1-(1+B\$4)^-E\$4)	=F56-C57

	H	I	J
43	T	Cflow	Saldi
44	=B44		0
45	=B45	=E45-E13	=J44*((1+B\$5)*(J44>=0)+(1+B\$6)*(J44<0))+I45
46	=B46	=E46-E14	=J45*((1+B\$5)*(J45>=0)+(1+B\$6)*(J45<0))+I46
47	=B47	=E47-E15	=J46*((1+B\$5)*(J46>=0)+(1+B\$6)*(J46<0))+I47
48	=B48	=E48-E16	=J47*((1+B\$5)*(J47>=0)+(1+B\$6)*(J47<0))+I48
49	=B49	=E49-E17	=J48*((1+B\$5)*(J48>=0)+(1+B\$6)*(J48<0))+I49
50	=B50	=E50-E18	=J49*((1+B\$5)*(J49>=0)+(1+B\$6)*(J49<0))+I50
51	=B51	=E51-E19	=J50*((1+B\$5)*(J50>=0)+(1+B\$6)*(J50<0))+I51
52	=B52	=E52-E20	=J51*((1+B\$5)*(J51>=0)+(1+B\$6)*(J51<0))+I52
53	=B53	=E53-E21	=J52*((1+B\$5)*(J52>=0)+(1+B\$6)*(J52<0))+I53
54	=B54	=E54-E22	=J53*((1+B\$5)*(J53>=0)+(1+B\$6)*(J53<0))+I54
55	=B55	=E55-E23	=J54*((1+B\$5)*(J54>=0)+(1+B\$6)*(J54<0))+I55
56	=B56	=E56-E24	=J55*((1+B\$5)*(J55>=0)+(1+B\$6)*(J55<0))+I56
57	=B57	=E57-E25	=J56*((1+B\$5)*(J56>=0)+(1+B\$6)*(J56<0))+I57

$$\sum_{h=0}^{20} T_2^{\max+W} \quad \sum_{t=1} \quad C_{h,t} = A_h R_t^{(1)} + (1 - A_h) R_t^{(2)} - R_t$$

	L	M	N	O	P	Q	R	S
42		0%	5%	10%	15%	20%	25%	30%
43	Tempi	Cash-flow						
44	1	-16450	-16450	-16450	-16450	-16450	-16450	-16450
45	2	-1758	-1342	-925	-509	-93	324	740
46	3	-1037	-621	-204	212	628	1045	1461
47	4	-322	94	511	927	1343	1760	2176
48	5	387	803	1220	1636	2052	2469	2885
49	6	1090	1506	1923	2339	2755	3172	3588
50	7	1787	2203	2620	3036	3452	3869	4285
51	8	2478	1780	1082	383	-315	-1013	-1711
52	9	3163	2465	1767	1068	370	-328	-1026
53	10	3842	3144	2446	1747	1049	351	-347
54	11	13965	13267	12569	11870	11172	10474	9776
55	12	13965	13267	12569	11870	11172	10474	9776
56	13	13965	13267	12569	11870	11172	10474	9776

	L	T	U	V	W	X	Y	Z
42		35%	40%	45%	50%	55%	60%	65%
43	Tempi	Cash-flow						
44	1	-16450	-16450	-16450	-16450	-16450	-16450	-16450
45	2	1156	1573	1989	2406	2822	3238	3655
46	3	1877	2294	2710	3127	3543	3959	4376
47	4	2592	3009	3425	3842	4258	4674	5091
48	5	3301	3718	4134	4551	4967	5383	5800
49	6	4004	4421	4837	5254	5670	6086	6503
50	7	4701	5118	5534	5951	6367	6783	7200
51	8	-2410	-3108	-3806	-4504	-5203	-5901	-6599
52	9	-1725	-2423	-3121	-3819	-4518	-5216	-5914
53	10	-1046	-1744	-2442	-3140	-3839	-4537	-5235
54	11	9077	8379	7681	6983	6284	5586	4888
55	12	9077	8379	7681	6983	6284	5586	4888
56	13	9077	8379	7681	6983	6284	5586	4888

	L	AA	AB	AC	AD	AE	AF	AG
42		70%	75%	80%	85%	90%	95%	100%
43	Tempi	Cash-flow						
44	1	-16450	-16450	-16450	-16450	-16450	-16450	-16450
45	2	4071	4487	4904	5320	5736	6153	6569
46	3	4792	5208	5625	6041	6457	6874	7290
47	4	5507	5923	6340	6756	7172	7589	8005
48	5	6216	6632	7049	7465	7881	8298	8714
49	6	6919	7335	7752	8168	8584	9001	9417
50	7	7616	8032	8449	8865	9281	9698	10114
51	8	-7297	-7996	-8694	-9392	-10090	-10789	-11487
52	9	-6612	-7311	-8009	-8707	-9405	-10104	-10802
53	10	-5933	-6632	-7330	-8028	-8726	-9425	-10123
54	11	4190	3491	2793	2095	1397	698	0
55	12	4190	3491	2793	2095	1397	698	0
56	13	4190	3491	2793	2095	1397	698	0

	L	M	N
42	0		=M\$42+\$B\$9
43	Tempi		
44	=B45	=M\$42*\$E29+(1-M\$42)*\$E45-\$E13	=N\$42*\$E29+(1-N\$42)*\$E45-\$E13
45	=B46	=M\$42*\$E30+(1-M\$42)*\$E46-\$E14	=N\$42*\$E30+(1-N\$42)*\$E46-\$E14
46	=B47	=M\$42*\$E31+(1-M\$42)*\$E47-\$E15	=N\$42*\$E31+(1-N\$42)*\$E47-\$E15
47	=B48	=M\$42*\$E32+(1-M\$42)*\$E48-\$E16	=N\$42*\$E32+(1-N\$42)*\$E48-\$E16
48	=B49	=M\$42*\$E33+(1-M\$42)*\$E49-\$E17	=N\$42*\$E33+(1-N\$42)*\$E49-\$E17
49	=B50	=M\$42*\$E34+(1-M\$42)*\$E50-\$E18	=N\$42*\$E34+(1-N\$42)*\$E50-\$E18
50	=B51	=M\$42*\$E35+(1-M\$42)*\$E51-\$E19	=N\$42*\$E35+(1-N\$42)*\$E51-\$E19
51	=B52	=M\$42*\$E36+(1-M\$42)*\$E52-\$E20	=N\$42*\$E36+(1-N\$42)*\$E52-\$E20
52	=B53	=M\$42*\$E37+(1-M\$42)*\$E53-\$E21	=N\$42*\$E37+(1-N\$42)*\$E53-\$E21
53	=B54	=M\$42*\$E38+(1-M\$42)*\$E54-\$E22	=N\$42*\$E38+(1-N\$42)*\$E54-\$E22
54	=B55	=M\$42*\$E39+(1-M\$42)*\$E55-\$E23	=N\$42*\$E39+(1-N\$42)*\$E55-\$E23
55	=B56	=M\$42*\$E40+(1-M\$42)*\$E56-\$E24	=N\$42*\$E40+(1-N\$42)*\$E56-\$E24
56	=B57	=M\$42*\$E41+(1-M\$42)*\$E57-\$E25	=N\$42*\$E41+(1-N\$42)*\$E57-\$E25

$$\prod_{h=0}^{20} S_{h,0} = 0, \quad \prod_{h=0}^{20} \prod_{t=1}^{T_2 \max + w} S_{h,t} = S_{h,t-1} \cdot \begin{cases} (1+i_3) & \text{se } S_{h,t-1} \geq 0 \\ (1+i_4) & \text{se } S_{h,t-1} < 0 \end{cases} + Cf_{h,t}$$

$$A_{\hat{h}} \Leftrightarrow S_{\hat{h}, T_2 \max + 1} = \max_h (S_{h, T_2 \max + 1})$$

	L	M	N	O	P	Q	R	S
57	Tempi	Saldi progressivi del c/c bancario						
58	0	0	0	0	0	0	0	0
59	1	-16450	-16450	-16450	-16450	-16450	-16450	-16450
60	2	-20182	-19766	-19349	-18933	-18517	-18100	-17684
61	3	-23641	-22758	-21875	-20993	-20110	-19227	-18345
62	4	-26800	-25395	-23990	-22585	-21180	-19775	-18370
63	5	-29628	-27639	-25649	-23659	-21669	-19679	-17689
64	6	-32094	-29449	-26804	-24159	-21514	-18869	-16224
65	7	-34158	-30779	-27400	-24022	-20643	-17264	-13886
66	8	-35779	-32693	-29607	-26521	-23435	-20349	-17263
67	9	-36909	-34151	-31393	-28635	-25877	-23119	-20361
68	10	-37496	-35106	-32715	-30324	-27934	-25543	-23152
69	11	-28031	-26051	-24072	-22093	-20113	-18134	-16155
70	12	-17429	-15911	-14392	-12874	-11355	-9836	-8318
71	13	-5556	-4553	-3551	-2548	-1546	-543	459

	L	T	U	V	W	X	Y	Z
57	Tempi	Saldi progressivi del c/c bancario						
58	0	0	0	0	0	0	0	0
59	1	-16450	-16450	-16450	-16450	-16450	-16450	-16450
60	2	-17268	-16851	-16435	-16018	-15602	-15186	-14769
61	3	-17462	-16579	-15697	-14814	-13932	-13049	-12166
62	4	-16965	-15560	-14155	-12750	-11345	-9941	-8536
63	5	-15699	-13710	-11720	-9730	-7740	-5750	-3760
64	6	-13579	-10934	-8289	-5644	-2999	-354	2291
65	7	-10507	-7128	-3749	-371	3008	6387	9536
66	8	-14177	-11091	-8006	-4920	-2135	614	3128
67	9	-17603	-14845	-12087	-9329	-6908	-4590	-2724
68	10	-20761	-18371	-15980	-13589	-11576	-9678	-8286
69	11	-14176	-12196	-10217	-8238	-6681	-5253	-4392
70	12	-6799	-5281	-3762	-2244	-1198	-298	-32
71	13	1462	2465	3467	4470	4942	5253	4852

	L	AA	AB	AC	AD	AE	AF	AG
57	Tempi	Saldi progressivi del c/c bancario						
58	0	0	0	0	0	0	0	0
59	1	-16450	-16450	-16450	-16450	-16450	-16450	-16450
60	2	-14353	-13937	-13520	-13104	-12688	-12271	-11855
61	3	-11284	-10401	-9518	-8636	-7753	-6870	-5988
62	4	-7131	-5726	-4321	-2916	-1511	-106	1299
63	5	-1770	219	2209	4199	6189	8179	10039
64	6	4936	7559	10005	12451	14897	17343	19657
65	7	12651	15742	18654	21565	24476	27388	30164
66	8	5606	8062	10333	12604	14875	17147	19280
67	9	-894	912	2530	4149	5767	7386	8864
68	10	-6935	-5701	-4749	-3796	-2844	-1891	-1082
69	11	-3578	-2894	-2526	-2157	-1789	-1420	-1212
70	12	183	250	-36	-321	-607	-892	-1358
71	13	4376	3746	2753	1735	717	-301	-1520

	L	M	N
57	Tempi		
58	0	0	0
59	=B45	=M58*((1+\$B\$5)^(M58>=0)+(1+\$B\$6)^(M58<0))+M44	=N58*((1+\$B\$5)^(N58>=0)+(1+\$B\$6)^(N58<0))+N44
60	=B46	=M59*((1+\$B\$5)^(M59>=0)+(1+\$B\$6)^(M59<0))+M45	=N59*((1+\$B\$5)^(N59>=0)+(1+\$B\$6)^(N59<0))+N45
61	=B47	=M60*((1+\$B\$5)^(M60>=0)+(1+\$B\$6)^(M60<0))+M46	=N60*((1+\$B\$5)^(N60>=0)+(1+\$B\$6)^(N60<0))+N46
62	=B48	=M61*((1+\$B\$5)^(M61>=0)+(1+\$B\$6)^(M61<0))+M47	=N61*((1+\$B\$5)^(N61>=0)+(1+\$B\$6)^(N61<0))+N47
63	=B49	=M62*((1+\$B\$5)^(M62>=0)+(1+\$B\$6)^(M62<0))+M48	=N62*((1+\$B\$5)^(N62>=0)+(1+\$B\$6)^(N62<0))+N48
64	=B50	=M63*((1+\$B\$5)^(M63>=0)+(1+\$B\$6)^(M63<0))+M49	=N63*((1+\$B\$5)^(N63>=0)+(1+\$B\$6)^(N63<0))+N49
65	=B51	=M64*((1+\$B\$5)^(M64>=0)+(1+\$B\$6)^(M64<0))+M50	=N64*((1+\$B\$5)^(N64>=0)+(1+\$B\$6)^(N64<0))+N50
66	=B52	=M65*((1+\$B\$5)^(M65>=0)+(1+\$B\$6)^(M65<0))+M51	=N65*((1+\$B\$5)^(N65>=0)+(1+\$B\$6)^(N65<0))+N51
67	=B53	=M66*((1+\$B\$5)^(M66>=0)+(1+\$B\$6)^(M66<0))+M52	=N66*((1+\$B\$5)^(N66>=0)+(1+\$B\$6)^(N66<0))+N52
68	=B54	=M67*((1+\$B\$5)^(M67>=0)+(1+\$B\$6)^(M67<0))+M53	=N67*((1+\$B\$5)^(N67>=0)+(1+\$B\$6)^(N67<0))+N53
69	=B55	=M68*((1+\$B\$5)^(M68>=0)+(1+\$B\$6)^(M68<0))+M54	=N68*((1+\$B\$5)^(N68>=0)+(1+\$B\$6)^(N68<0))+N54
70	=B56	=M69*((1+\$B\$5)^(M69>=0)+(1+\$B\$6)^(M69<0))+M55	=N69*((1+\$B\$5)^(N69>=0)+(1+\$B\$6)^(N69<0))+N55
71	=B57	=M70*((1+\$B\$5)^(M70>=0)+(1+\$B\$6)^(M70<0))+M56	=N70*((1+\$B\$5)^(N70>=0)+(1+\$B\$6)^(N70<0))+N56

